

# The Political Economy of Immigration, Investment, and Naturalization<sup>1</sup>

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This version: June 2nd, 2021

ABSTRACT: This paper provides the first economics-based rationale for the purpose of naturalization. It presents a new political-economy model of immigration featuring a hold-up problem between the government and capital owners over immigration policy, that causes under-investment in capital. Naturalization plays the role of an institution that the government can use to ‘tie its hands’ to the presence of naturalized immigrants, partially resolving the hold-up problem. The model is used to explain the Koopmans-Michalowski paradox: that while dictatorships are more open in terms of policies towards immigrants, democracies are more open in terms of extending immigration rights through naturalization.

KEYWORDS: hold-up problem, immigration policy, institution, migration, naturalization

JEL CLASSIFICATION NUMBERS: D02, F22, J61, O43, P16

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<sup>1</sup>For useful comments and conversations about earlier drafts, we are grateful to numerous friends and colleagues, as well as seminar participants at Cal Poly, Cardiff, Exeter, a Lisbon Conference on Game Theory and Applications, and an InsTED workshop at Nottingham. Ghosh gratefully acknowledges funding from an ESRC Southwest Doctoral College PhD scholarship.

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# 1 Introduction

A nascent literature in economics shows that the political economy of lobbying plays an important role in the determination of immigration policy. Facchini and Willmann (2005) model, as a menu auction, the way that politically organized groups use financial contributions to influence the level of immigration policy.<sup>4</sup> Facchini, Mayda and Mishra (2011) present the first empirical evidence on the relationship between lobbying and immigration policy based on data for the US. Their estimates suggest that a 10% increase in the size of lobbying expenditures by business groups, per native worker, is associated with a 3.1-5% larger number of visas per native worker, while a one percentage-point increase in the union membership rate is associated with a 2.6-5.6% lower number of visas per native worker.

The aim of our paper is to rationalize naturalization as an institution in terms of the political economy of immigration policy. Naturalization refers to the process through which an immigrant gains citizenship of a country. The main rights gained from citizenship are the rights to live and work in that country indefinitely.<sup>5</sup> By contrast, an immigrant who is not naturalized can only live and work in a country of which they are non-native for a limited period determined by a visa granted by the government.<sup>6</sup> While the economics literature on immigration is large, it has paid very little attention to naturalization. The literature on naturalization that does exist in economics takes a case-studies approach to understand its costs and benefits in specific countries (Bevelander and DeVoretz 2008). Therefore, to the best of our knowledge, our paper provides the first political-economy based rationalization of naturalization.

Our consideration of naturalization from a political-economy standpoint entails examining a government's incentive to set immigration policy, taking into account its own interests as well as those of its citizens. In thinking of naturalization as an institution, we will follow North's (1981) approach, whereby an institution imposes constraints on policymakers in terms of how they set a particular policy. In our case, we think of naturalization as an institution that imposes a constraint

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<sup>4</sup>In the same paper, Facchini and Willmann also consider the effect of lobbying on government policy for capital mobility.

<sup>5</sup>Citizenship is the legal institution that designates full membership of a state, with associated rights and duties. In addition to the right to live and work, in a democracy a citizen also gains the right to vote, and legal protection in the case of criminal charges. There are three main ways to gain citizenship: by birth, by marriage, and by naturalization. In this paper we will only consider citizenship by naturalization. See Bertocchi and Strozzi (2010) on the political-economic determinants of citizenship rights by birth.

<sup>6</sup>Some visas only allow an immigrant to live in a country and not to work, usually as a dependent of another person. Our concern here will be solely with the right to live and work in a country.

on the way that the government sets immigration policy.

Crucial for our political-economy focus is that while the government can control the number of non-naturalized immigrants present in a country through the issuance of visas, it cannot reduce the number of naturalized immigrants (once they have been naturalized) because they have the right to remain in the country indefinitely. Hence, we can think of naturalization as an institution through which the government ties its hands to the presence of some immigrants in the country.

There is a literature on naturalization in political science that takes an institutional perspective, and this sets the frame for our paper. Koopmans and Michalowski (2017) observe that the form of government, democracy or dictatorship, plays a decisive role in determining whether a country will extend naturalization rights to immigrants. They present evidence that democracies tend to be *less* restrictive than dictatorships in terms of the naturalization rights that they extend to immigrants. However, surprisingly, democracies tend to be *more* restrictive than dictatorships in terms of the immigration policies that they set. Koopmans and Michalowski (2017) say that, as yet, we have no explanation for these apparently inconsistent approaches between dictatorships and democracies to naturalization rights on the one hand and immigration policy on the other.

For suggestive evidence of this apparent inconsistency, compare the immigration policies on the one hand, and naturalization rights on the other, of a democracy like the United States of America (USA) to a dictatorship like the United Arab Emirates (UAE).<sup>7</sup> The UAE has the highest proportion of international migrants among all the countries of the world. According to the Migration Data Portal, UAE has an international migration stock of 87.9% as a result of its very relaxed policies towards immigration. By contrast, the USA's international migration stock is a comparatively small 15.4%, indicating a comparatively restrictive approach towards immigration policy.<sup>8</sup> However, the UAE is extremely stringent in its approach to granting immigrants the right to naturalize. The USA, on the other hand, has a relatively open and transparent procedure for extending naturalization rights to immigrants.

FitzGerald & Cook-Martin (2014) present econometric evidence that immigration policy tends to be more open in dictatorships than in democracies. Koopmans and Michalowski's (2017) review of FitzGerald & Cook-Martin (2014) provides the following political-economy based explanation for

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<sup>7</sup>According to United Nations Migration (2020), published prior to the global pandemic of Covid-19 which has resulted in a sharp curtailment of immigration in 2020-21, there has been a steady increase in the volume of international migrants in the world. The number of immigrants was estimated to be around 272 million in 2019, with about two thirds being labor migrants. Most of this migration was from developing countries to developed countries like the UAE and USA.

<sup>8</sup>Numbers obtained from Migration Data Portal.

this finding. Economic elites, who tend to be firm- and capital-owners benefit from cheaper labor because this increases their income. A feature of dictatorships is that the elite have a disproportionate influence over economic policy.<sup>9</sup> If the elite own capital, then in a dictatorship they will use their influence to determine a relatively open immigration policy that leads to relatively high immigration and hence cheap labor. By contrast, a democracy is more likely to have a relatively closed immigration policy that leads to lower immigration and higher wages, reflecting the interests of workers.<sup>10</sup>

Following this line of argument, one might have predicted that dictatorships would be more likely to extend naturalization rights to immigrants than democracies would because this would further facilitate immigration and investment. But, as already noted above, the opposite appears to be the case in practice. This leads Koopmans and Michalowski (2017) to acknowledge an apparent inconsistency between democracies and dictatorships over the way that immigration policy is operated and naturalization rights extended. We will refer to this apparent inconsistency as ‘the Koopmans-Michalowski paradox’.

It is not obvious how an economics perspective can provide a resolution to the Koopmans-Michalowski paradox. We have seen from Facchini et al (2011) that the US government derives benefits from lobbying over immigration policy. From this, it is natural to ask why any government, democracy or dictatorship, would want to foreclose lobbying over immigration by granting naturalization rights to any immigrant. Our model of naturalization as an institution can explain both why a government would foreclose lobbying through naturalization, and why democracies are more likely to benefit from this institution than dictatorships are. Hence, we are able to resolve the Koopmans-Michalowski paradox.

Our model features a single small country with one sector and two factors of production: labor and capital. The population is divided into three groups: workers, capital owners and the government. Workers earn their marginal product and increased immigration lowers their wages, as per the textbook competitive model. This is because the ‘world wage’ is lower than in the country we are

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<sup>9</sup>In the terminology of Bueno de Mesquita, Smith, Siverson and Morrow (2005), the government are chosen by ‘the selectorate’. In a democracy, the selectorate equates to the whole of the adult population. In a dictatorship, the selectorate equates to a small privileged subsection of the population commonly referred to as ‘the elite’. It is the elite’s power over the selection of government that gives them a disproportionate influence over economic policy.

<sup>10</sup>This is not to say that all dictatorships are necessarily more open to immigration than all democracies. We will adopt Olson’s (2000) classification of dictatorships into capitalist and communist dictatorships. Using this classification, we will be able to show using our model why capitalist dictatorships like the UAE set more open immigration policy than communist dictatorships like China. The underlying assumption is that communist dictatorships tend to reflect the interests of workers rather than capitalists. Interpreted in these terms, Fitzgerald and Cook-Martin’s (2014) finding that dictatorships tend to set more open immigration policy than democracies suggests that dictatorships tend to be capitalist.

considering and provides the motivation for immigration. The capital owners are firm - as well as capital - owners. This means they earn profits as well as rental returns on capital. They decide how much capital to invest in production and benefit from having more immigrant labor, as a higher labor supply lowers the cost of production through lower wages. The government controls immigration by setting an immigration quota.

Based on the structure of our model, the first contribution of this paper is to provide the first economics-based characterization of naturalization. We show that, surprisingly, naturalization is not necessarily good for social welfare. We do this in the context of the planner's problem, by allowing the planner to choose the naturalization rate of immigrants on behalf of the country we are considering. We show that (national) social welfare may be monotonically increasing or monotonically decreasing in the naturalization rate, depending on the world wage. When the world wage is sufficiently low, the positive effect of naturalization on increased profits tends to dominate and social welfare is monotonically increasing in the naturalization rate. Conversely, when the world wage is sufficiently high, the negative effect of naturalization on the domestic wage tends to dominate and social welfare is monotonically decreasing in the naturalization rate.

It is surprising that naturalization may be bad for social welfare if we think of a commitment to naturalization as being analogous to a commitment to trade liberalization, say through a trade agreement.<sup>11</sup> A trade agreement enables the government to commit to trade liberalization, increasing social welfare by removing the distortion created by trade policy. A natural expectation might be that naturalization serves the same purpose by putting at least some immigrants beyond the reach of distortionary immigration policy.

The fact that naturalization does not necessarily increase social welfare has to do with two additional features of naturalization not exhibited by trade liberalization through a trade agreement. First, naturalization increases the labor supply, which lowers wages and hurts workers who are in the majority of the population. Second, while naturalization at least partially removes a distortion created by immigration policy, it also leaves in place a second distortion created by the fact that the government typically collects a share of the quota rents that would otherwise go to immigrants. So a commitment to naturalization has welfare-reducing effects that may overwhelm the beneficial effects of the removal of a distortionary policy.

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<sup>11</sup>Trade policy and immigration policy are regarded as substitutes in the sense that immigration involves the movement of labor directly, while international trade involves the movement of labor indirectly, embodied in the goods that are traded.

Next, based on a two-stage game, we show that the government faces a hold-up problem over immigration policy. In Stage 1, capital owners fix the level of capital. In stage 2, the government sets the immigration quota. We set up the model so that the quota is fully filled. Importantly, in this scenario we initially assume that lobbying is not allowed. We refer to this scenario as ‘government with no lobbying’. In Stage 1, the capital owners undertake investment, whereby the profit-maximizing level of capital formation is inefficiently low. This is because they anticipate that, in Stage 2, the government will have an incentive to extract some of the returns from investment using the immigration quota.<sup>12</sup>

Hence, in equilibrium, capital owners scale back investment in Stage 1, in anticipation of being held up in Stage 2. Otherwise, the forces at work are the same as described above for the planner’s problem. The end result is that, in equilibrium, the immigration quota is tighter than the efficient level, the wage is at the efficient level, and investment is sub-optimally low. Thus we characterize the way that the government holds up capital owners over investment in our model.

Then we show that naturalization mitigates the hold-up problem. The reason is that naturalization places beyond the government’s control the presence of a certain number of immigrants in the country, and this encourages investment. As a result, the government may be better off committing to naturalization, thus preventing the long-run distortion of under-investment.

To examine the way that naturalization mitigates the hold-up problem, we impose a positive naturalization rate of immigrants on the government. Unlike the social planner, the government is assumed not to be able to determine the naturalization rate, which it simply takes as given. In addition, we do not think of the government as choosing whether to adopt naturalization. We assume naturalization is imposed on the government and then examine its implications. This reflects our idea that, where a government is constrained by the institution of naturalization, its rules are enshrined in the country’s constitution and the rule of law ensures that the government must abide by them.

Nevertheless, we examine the implications of naturalization for the government in a way that parallels our analysis of naturalization for the social planner. We consider the effect of increasing

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<sup>12</sup>The fully general definition of the hold-up problem is as follows: “If an investor receives less than 100% of the returns, he or she will invest too little from the perspective of maximizing total wealth or well-being ... because a party other than the investing party can take actions to capture some of the returns generated” Hermalin (2010). The hold-up problem is often associated with a restricted situation where the actions that the other party can take involve a bargain with the investing party. See for example Hermalin and Katz (2009). Ultimately, we will consider ‘government with lobbying,’ where bargaining between capital owners and the government over immigration policy gives rise to hold-up. However, initially we consider ‘government with no lobbying,’ which accords with the more general definition of hold-up, whereby government action involves setting an immigration quota to capture some of the returns from investment, but does not bargain with capital owners.

the naturalization rate from zero to some positive level. In direct parallel to the planner's solution, we find that an increase in the naturalization rate either monotonically increases or monotonically decreases the government's payoff, depending on whether the world wage is sufficiently low or sufficiently high respectively.

The paper's second contribution is to provide a resolution of the Koopmans-Michalowski paradox. To motivate the Koopmans-Michalowski paradox, we need to distinguish between dictatorship and democracy. Our approach adopts Olson's (2000) classification of dictatorships into capitalist dictatorships and communist dictatorships. By definition, a capitalist dictatorship is one where the elite are comprised of owners of capital, while a communist dictatorship is one where the elite represent the interests of workers.

Fitzgerald and Cook-Martin's (2014) finding that dictatorships tend to set more open immigration policy than democracies suggests that dictatorships tend to be capitalist. This is because more open immigration policy tends to push down wages, hurting workers but increasing the income of capital owners. Therefore, for consistency with Fitzgerald and Cook-Martin's (2014) finding, we will assume that a dictatorship is capitalist unless explicitly stated otherwise.<sup>13</sup>

Under this assumption, and government with no lobbying, the only distinction between democracy and dictatorship is that a dictatorship places a greater weight on the income of capital owners. The model predicts that democracies tend to be more restrictive than dictatorships, both in terms of immigration policy, and in terms of naturalization rights. In terms of both immigration policy and rights, the dictatorship's payoff is influenced by the greater weight that it places on the income of capital owners. As a result, a dictatorship favors more open immigration policy than the democracy would, as well as naturalization because this further facilitates immigration and investment. Therefore, under the assumption of government with no lobbying, we cannot resolve the Koopmans-Michalowski paradox.

We then explore the idea that lobbying of the government by capital owners over immigration policy provides the missing link through which we can resolve the Koopmans-Michalowski paradox. To do this, we extend our framework to allow capital owners to coordinate their influence over immigration policy through a lobby. The government's objective function is defined in terms of a weighted average of social welfare and lobby contributions, as in Facchini and Willmann (2005) and following Grossman and Helpman (1994). We call this 'government with lobbying'.

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<sup>13</sup>While we adopt as our baseline a specification where dictatorships are characterized as capitalist, we will be able to use our model to characterize the behavior of communist dictatorships as well.

Under government with lobbying, Stage 1 is the same as in government with no lobbying. Stage 2 is different, in that we introduce lobbying of the government by capital owners as a Nash bargain, whereby their joint surplus is maximized to determine the level of the immigration quota. Following that, the surplus that arises from lobbying is shared between the government and the lobby according to their given bargaining powers. This consequently leads to the determination of the lobby's optimal contribution schedule. From this, given the capital stock, the amount of immigration and lobby contributions are determined. To consider naturalization, as in the case of government with no lobbying, we impose the constraint on the government to naturalize a number of immigrants.

A crucial assumption that we make when we introduce lobbying is that a democratic government has weaker bargaining power vis-a-vis the lobby than a dictatorship has. To substantiate this assumption, we argue that democratic governments have their hands tied by other institutions, with the rule of law being an umbrella institution that arches over several others. Hence, democratic governments are more constrained in their dealings with lobby groups by the rule of law, and in that sense their bargaining power is weaker vis-a-vis the lobby than is the bargaining power of dictatorships, who tend not to be constrained by the rule of law. We use this logic to support our assumption that, in the Nash bargain over immigration policy, a democracy has weaker bargaining power vis-a-vis the lobby than a dictatorship has.

With this framework in place, we are able to resolve the Koopmans-Michalowski paradox. To do so, we show that only in countries where government bargaining power against lobbies is relatively weak will the institution of naturalization be beneficial to the government. From our assumption that democracies have relatively weak bargaining power, it follows that under government with lobbying, only democracies will benefit from the institution of naturalization.

The reason is that when a government has weak bargaining power, as in a democracy, the revenues it gains from lobbying are relatively small compared to the distortion costs of under-investment. So it is worth the government foreclosing some lobbying over immigration policy through naturalization in exchange for the additional investment that results. Where government bargaining power is relatively strong, as is more likely in a dictatorship, a government will be worse off under the constraint of naturalization because of the relatively large lobby revenues foregone. Hence we are able to resolve the Koopmans-Michalowski paradox. Importantly, we are also able to show that naturalization under democracy enhances economic efficiency, while the dictatorship's gain from an absence of naturalization goes hand in hand with a reduction in economic efficiency.



Our paper contributes to the literature on the political economy of international economic policy, because the tensions created by immigration policy are different from those examined in the past literature. Much of this literature focuses on international trade policy, and it is valuable to consider the extent to which the insights from that literature extend to other areas of international economic policy such as immigration policy.<sup>14</sup> Our work brings to light an important difference between trade policy and immigration policy. With trade policy, the incentive for the government to set the efficient economic policy of free trade is aligned with that of citizens through the usual argument for the gains from trade. Therefore, lobbying is actually critical for the existence of the hold-up problem in the sense that in the absence of a lobby the government would adopt the efficient policy of free trade. With immigration policy, even in the absence of lobbying, the government is drawn away from the efficient policy by workers as voters who favor protection from immigration. As a result, a hold-up problem exists with immigration policy even in the absence of lobbying. Moreover, in the setting of immigration, lobbying is actually in favor of the efficient solution of more open immigration policy. So the tensions that the government typically faces over immigration policy are the reverse of what it typically faces over trade policy.

Given the contrast in the tensions that the government faces between trade policy and immigration policy, it is actually surprising that the commitment-based logic for an international trade agreement carries over to the logic for naturalization as well. Although our model is original, we borrow from Maggi and Rodriguez-Clare (1998, henceforth MR-C) the feature that capital is fixed before lobbying takes place. Their model provides a rationale for an international trade agreement that ties the government's hands to free trade, based on the hold-up problem. Similarly, we have a rationale for naturalization that ties the government's hands to a certain level of openness to immigration, again based on the hold-up problem.

This raises the possibility that the logic may generalize further still, to other settings where the tensions are in line with those of immigration policy and the opposite of international trade policy. We also show that MR-C's insight, that a weak government gains from tying its hands to a trade agreement while a strong government will not, extends to the context of immigration policy. Our work suggests that this insight, too, may generalize to other settings. We provide an example and discuss this further in the concluding section. Moreover, while MR-C do not provide any basis for the strength of governments' bargaining power, we link this to whether the government is a democracy

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<sup>14</sup>See Rodrik (1995), Helpman (1997), Gawande and Krishna (2003), and Maggi (2014) for a comprehensive set of surveys of the trade policy literature.

or a dictatorship. This step is crucial in enabling us to resolve the Koopmans-Michalowski paradox, which is an entirely original contribution of our work.

Our paper also contributes to the literature on the relationship between political and economic institutions. In terms of political institutions, there is broad agreement that while dictatorship can be good for economic growth, it is often detrimental to equality and welfare broadly defined. At the same time, it is agreed that while democracy is no panacea, it is generally associated with a greater likelihood of economic development and improved well-being across society. Democracy, when it functions well, is an inclusive political institution that underpins good economic institutions because they tend to be inclusive as well, and hence more effective in supporting economic development. Conversely, dictatorship enables the elite to monopolize political power and hence structure economic institutions not to support economic development but instead to maximize the rents that they capture (Acemoglu, Johnson and Robinson 2005).

A new observation that follows from our framework is that when a government is constrained by the rule of law overall, as it is under democracy, it is more likely to benefit from being constrained in yet another way. Our analysis implies that if the government is constrained by the rule of law, it is more likely to benefit from being constrained in its choices over immigration policy. This insight suggests that inclusive institutions beget other inclusive institutions.

Moreover, our paper sits at the intersection of the three literatures mentioned above. As already mentioned, we believe we are the first to argue that a key feature of the political economy of immigration policy, naturalization, has an institutional interpretation. At the same time, our institutional interpretation of naturalization has yielded an economics-based rationalization of why democracies and dictatorships take differing approaches to immigration policy and naturalization rights. Finally, locating the discussion around the form of government, democracy and dictatorship, provides a link between political institutions and the economic institution of naturalization.

The paper is organized in the following way. Section 2 sets out the analytical framework. Section 3 outlines the benchmark social planner's problem. Section 4 examines government with no lobbying, motivating the Koopmans-Michalowski paradox. Section 5 then introduces lobbying to the model, and shows that doing so enables us to resolve the Koopmans-Michalowski paradox. Section 6 concludes. Most proofs of propositions are in the body of the paper, but additional details are in the Appendix where required.

## 2 The Model

The economy consists of one sector producing a single homogeneous final good in quantity  $Q$ , with two factors of production, labor,  $L$ , and capital,  $K$ . We consider a production function of the form  $Q = K^\alpha \sqrt{L}$  such that  $\alpha + \frac{1}{2} < 1$ .<sup>15</sup> Thus, the production function exhibits decreasing returns to scale.<sup>16</sup> The economy consists of a fixed number of domestic workers and capital owners. The ‘world wage’ is the wage prevailing outside of the economy, and is a parameter fixed at  $w^*$ . There is an infinite supply of immigrants to the economy as long as immigrants get a higher wage there than the world wage. Capital owners are immobile by assumption. Each capital owner owns the capital invested in her firm, as well as the firm itself. Firms are competitive and they are price takers in the goods and factor markets. Factor markets are competitive, so labor and capital are paid their marginal products:  $w = \frac{1}{2} \frac{K^\alpha}{\sqrt{L}}$  and  $r = \alpha K^{\alpha-1} \sqrt{L}$  respectively, where  $w$  denotes the wage and  $r$  denotes the interest rate.

The mass of the domestic labor force is normalized to 1, while that of the capital owners is normalized to  $s$ , so the total mass of the native born population in the economy is  $1 + s$ . We will denote by  $l_I$  the domestic quota of immigrant labor, and we will set the world wage sufficiently low that the quota is filled for any  $l_I$ . Because all immigrants are assumed to be workers, the labor force is given by  $L = L(l_I) = 1 + l_I$ . We assume that native workers and immigrants are perfect substitutes for each other in the labor force. Therefore, entry of more immigrant labor will compete down the wage, all else equal, and thus hurts native workers.

Assuming the final good is the numeraire, and capital owners are homogeneous, gross profits are given by

$$\pi = K^\alpha \sqrt{L} - rK - wL. \quad (1)$$

Note that a lower wage increases profits of the capital owners. So there is a tension in the model between workers and capital owners over immigration and the wage.

The immigration quota,  $l_I$ , plays a key role in the determination of income of both workers and capital owners. We can already see from the fact that  $l_I$  helps determine  $L$ , and  $L$  helps determine  $w$ , that  $w$  is a function of  $l_I$ . We will refer to  $w$  as the ‘domestic wage’, to distinguish it from the world

<sup>15</sup>The main results of the model hold for a more general production function of the form  $Q = K^\alpha L^\beta$  where  $\alpha + \beta < 1$ . Here we are assuming  $\beta = \frac{1}{2}$  for clarity of exposition.

<sup>16</sup>Since firms are competitive, the production function should exhibit non-increasing returns to scale. Assuming decreasing returns is without loss of generality. It is akin to assuming a production function that exhibits constant-returns-to-scale in labor and capital, plus managerial ability as a fixed factor that is distributed evenly among the capital owners. The net returns after paying labor and capital accrue entirely to the fixed factor.

wage,  $w^*$ . We will write the function for the domestic wage as  $w(l_I)$ .

Regarding profits, substituting the above expressions for  $L$ ,  $r$ , and  $w$  into the expression for gross profits, (1), and thereby recognizing that  $\pi$  is a function of  $l_I$ , we obtain

$$\begin{aligned}\pi(l_I) &= K^\alpha \sqrt{L(l_I)} - \alpha K^{\alpha-1} \sqrt{L(l_I)} K - \frac{1}{2} \frac{K^\alpha}{\sqrt{L(l_I)}} L(l_I) \\ &= \left( \frac{1}{2} - \alpha \right) K^\alpha \sqrt{L(l_I)}.\end{aligned}\tag{2}$$

From this expression we can see that the decreasing-returns assumption, which implies  $\alpha < \frac{1}{2}$ , allows profits for capital owners. Assuming a unit cost of installing capital, and recognizing that  $r$  is also a function of  $l_I$ , the function for net income of capital owners,  $\Pi$ , is given by:

$$\begin{aligned}\Pi(l_I) &= \pi(l_I) + r(l_I)K - K \\ &= \frac{1}{2} K^\alpha \sqrt{L(l_I)} - K.\end{aligned}\tag{3}$$

Turning to immigrant income now, for tractability we will assume that each immigrant captures a fixed amount of the quota rent, which we will treat as a non-negative parameter,  $\gamma$ . Accordingly, we will refer to  $w^* + \gamma$  as the ‘immigrant wage’. The government therefore captures  $(w(l_I) - w^* - \gamma)$  as quota rent per immigrant.<sup>17</sup>

We will leave for later the determination of  $K$  since that is affected fundamentally by the regime: the planner, or the government, and in turn whether the government is a dictatorship or a democracy, and whether or not they can receive lobby contributions. For now it suffices simply to recognize that  $l_I$  plays a role in determining the income both of capital owners and of workers, as well as the government itself through the quota rent.

## 2.1 Naturalization

We define  $\phi$  as the proportion of immigrants whom the country naturalizes, where  $\phi = 0$  represents no naturalization, and  $\phi = 1$  represents that 100% of immigrants are naturalized. Rather than model

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<sup>17</sup>To be clear, immigrants and domestic workers earn the same wage,  $w(l_I)$ . However, because the government captures  $(w(l_I) - w^* - \gamma)$  as quota rent, the immigrant effectively earns  $w^* + \gamma$ . Quota rents collected by the government come in a variety of forms. For example, in the United Kingdom (UK) these include visa costs, and the National Health Service (NHS) surcharge payable by immigrants to the UK (from outside the European Union at the time of writing, and possibly later from European Union countries as well depending on the outcome of the Brexit process). The NHS surcharge introduced on April 6th, 2015 is £200 per year for temporary non-European Economic Area (non-EEA) migrants and £150 per year for non-EEA students. Revenue collected from the NHS surcharge between April 6th, 2015 - March 14th, 2016 by the UK government was £175.6m. Home Office income from visa and immigration revenue was £1086m, and £1182m for the years 2015-16 and 2016-17 respectively.

the process of naturalization, we think of  $\phi$  as capturing an average tendency to naturalize, or in other words a ‘naturalization rate’, and examine the implications of this on economic outcomes. By assumption, not all immigrants can naturalize. Accordingly,  $\phi \in [0, \bar{\phi}]$ , where  $\bar{\phi} \in (0, 1)$  is the upper bound to the naturalization rate. We think it reasonable to assume that factors extraneous to the model, such as fears that excessive immigration will harm national cultural identity, will mean a country would not want to naturalize all immigrants.

As in practice, once an immigrant is naturalized, the government treats them as a native. Even after naturalization, by assumption immigrants are workers and cannot become capital owners. This assumption preserves the tension in the model between native workers and capital owners over immigration and the domestic wage.

The larger the share of immigrants that are naturalized, the smaller the share from whom the government can collect quota rent. The total amount that it captures in quota rent is thus given by  $(1 - \phi)(w(l_I) - w^* - \gamma)l_I$ . Once an immigrant is naturalized, the government considers their welfare in terms of their wages, just as it would for native born workers. We will refer to the population of native born works and naturalized workers as domestic workers, to distinguish them from (unnaturalized) immigrants. Hence the term in the social welfare function that reflects the domestic wage,  $w(l_I)$ , will be weighted by the population of domestic workers,  $1 + \phi l_I$ .

## 2.2 The Social Welfare and Government Payoff Functions

The social welfare function is of standard utilitarian form:

$$SW(l_I) = s\Pi(l_I) + (1 + \phi l_I)w(l_I) + (1 - \phi)(w(l_I) - w^* - \gamma)l_I. \quad (4)$$

It is a weighted sum of capital owners’ and workers’ income, plus the income that the government derives from immigration quota revenue, where the weights reflect population shares of capital owners and workers respectively. To calculate the efficient solution, we assume that the social planner chooses  $\phi$ ,  $l_I$ , and  $K$  simultaneously to maximize  $SW(l_I)$ .

The government payoff function takes the form:

$$GW(l_I) = zs\Pi(l_I) + (1 + \phi l_I)w(l_I) + (1 - \phi)(w(l_I) - w^* - \gamma)l_I + ac(l_I)l_I. \quad (5)$$

The government payoff function modifies the social welfare function in two ways. First, it puts a weight  $z \geq 1$  on the term  $s\Pi(l_I)$ , and hence on the welfare of capital owners. This allows, in a reduced

form way, for the possibility that the government puts greater weight on the welfare of capital owners than workers. The approach of putting extra weight on the welfare of a particular group within society is well established in the trade policy literature.<sup>18</sup> Here we will use it to capture the idea that dictatorships tend to place more weight on the welfare of capital owners than democracies do. Concretely, we will say that while in a democracy the welfare of capital owners will receive the same weight as that of workers,  $z = 1$ , in a dictatorship they may receive additional weight  $z > 1$ .

Second, the government payoff function places a weight  $a > 0$  on lobby contributions. We define ‘ $a$ ’ as the degree of responsiveness of the government to lobby contributions. Capital owners can lobby the government for more immigration by making a financial contribution,  $c(l_I)l_I$ . In setting  $l_I$  to maximize  $GW(l_I)$ , the government balances the tension between workers and capital owners over domestic wages, against the financial contribution made by the capital owners through the lobby.

### 3 The Social Planner’s Problem

To maximize social welfare, (4), we assume that the planner can set the level of capital,  $K$ , and the immigration quota,  $l_I$ , but is constrained in being unable to allocate the level of employment directly. Also, in solving the planner’s problem, the planner sets the rate of naturalization,  $\phi \in [0, \bar{\phi}]$ . Since the planner is free to set  $\phi$ , naturalization does not impose a constraint on the planner in the way that it will on the government. Where relevant, a superscript- $SP$  will be used to denote the planner’s solution for a particular variable. The planner sets simultaneously the naturalization rate,  $\phi$ , the level of capital,  $K^{SP}$ , and the immigration quota,  $l_I^{SP}$ .

Taking  $\phi$  and  $K$  as given, and using the solutions for  $r$  and  $w$ , plus the expression for  $\pi$ , (1), we can solve for the socially efficient level of immigration,  $l_I^{SP}$ , that maximizes (4):

$$l_I^{SP} = \left( \frac{1}{2w^{SP}} \right)^2 K^{2\alpha} - 1, \quad (6)$$

where

$$w^{SP} = \frac{2(1 - \phi)(w^* + \gamma)}{1 + s}. \quad (7)$$

For  $w^* + \gamma > 0$  and  $s < 1 - 2\phi$ , the domestic wage is higher than the immigrant wage, which is necessary for positive immigration. To ensure that  $s > 0$ , without loss of generality we will restrict the domain of the upper bound of  $\phi$  further to  $\bar{\phi} \in (0, \frac{1}{2})$ . Since the parameter restriction  $s < 1 - 2\phi$  gives

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<sup>18</sup>This approach originated with Corden (1974) and has been a mainstay of the international trade policy literature ever since.

rise to positive immigration, we will impose it throughout our analysis of the planner's problem. (In the next section, where we relax the assumption that  $z = 1$ , we will modify the restriction on  $s$  to accommodate  $z > 1$ .) Crucially for our subsequent analysis, note that  $w^{SP}$  is increasing in  $w^* + \gamma$ , and tends to zero as  $w^* + \gamma$  tends to zero.

Using (7), equation (6) shows that the larger the mass of capital owners in the total population  $s$ , and the larger the amount of capital,  $K^{SP}$ , the higher will be the immigration quota,  $l_I^{SP}$ . The higher is the immigrant wage,  $w^* + \gamma$ , the lower will be  $l_I^{SP}$ , since the marginal gain in quota rent from tightening the immigration quota is reduced by higher  $w^* + \gamma$ . Also, the higher is the naturalization rate,  $\phi$ , the higher is  $l_I^{SP}$  simply to offset the loss of revenue from a higher rate of naturalization.

Maximization of the social welfare function (4) yields capital under the social planner as:

$$K^{SP} = \left( \frac{(1+s)\alpha}{2s} \frac{1}{2w^{SP}} \right)^{\frac{1}{1-2\alpha}}. \quad (8)$$

Equation (8) shows that  $K^{SP}$  is directly proportional to the share of capital required for production,  $\alpha$ , and  $s$ . However the higher is  $w^* + \gamma$ , the lower will be  $K^{SP}$  because this implies a higher domestic wage,  $w^{SP}$ , and hence a lower return to capital. It could be written in a more compact way, but we will write it like this to facilitate comparison to the level of  $K$  under other regimes.

Turning to profit under the social planner's problem, and using the above solutions for  $l_I^{SP}$  and  $w^{SP}$ , (6) and (7), in (2), we obtain

$$\pi^{SP} = \frac{(1-2\alpha)(K^{SP})^{2\alpha}}{4w^{SP}}. \quad (9)$$

From this we can see that  $\pi^{SP}$  is increasing in  $K^{SP}$ , but decreasing in  $w^{SP}$  and, underpinning that,  $w^* + \gamma$  as one would expect.

Next we consider the socially efficient level of net income of capital owners, given by  $\Pi(l_I^{SP})$ . Using (6) in (3), we obtain

$$\Pi^{SP} = \frac{(K^{SP})^{2\alpha}}{4w^{SP}} - K^{SP}. \quad (10)$$

Under the restriction  $s < 1 - 2\phi$ ,  $K^{SP}$  is less than the value of  $K$  that would maximize  $\Pi(l_I^{SP})$ . This stands to reason since the planner balances the welfare of capital owners against that of workers, and the value of  $K$  that would maximize  $\Pi(l_I^{SP})$  would correspond to an inefficiently low domestic wage. Therefore, the planner's solution features  $\Pi(l_I^{SP})$  increasing in  $K^{SP}$  and  $\alpha$ , and decreasing in  $w^{SP}$  and, underpinning that,  $w^* + \gamma$ , again as one would expect.

### 3.1 Naturalization

From the above solutions, we can see (using the expression for  $w^{SP}$ , 7) that an increase in the naturalization rate,  $\phi$ , increases  $l_I^{SP}$ ,  $K^{SP}$ , and  $\Pi^{SP}$ , but decreases  $w^{SP}$ , which may in turn decrease the total quota rent collected by the government,  $(1 - \phi)(w^{SP} - w^* - \gamma)l_I^{SP}$ . To see the overall effect, substitute equations (3) and (6)-(8) into (4), and simplify to obtain social welfare in reduced form:<sup>19</sup>

$$SW = \frac{1+s}{2}w^{SP} + \psi^{SP}K^{SP}, \quad (11)$$

where

$$\psi^{SP} = \frac{s(1-2\alpha)}{2\alpha} > 0.$$

Writing  $SW$  in this way highlights the tension between the element that is increasing in  $w^{SP}$ , and the element that is decreasing in  $w^{SP}$  via its effect on  $K^{SP}$ : recall equation (8). We can then see from (7) that  $\phi$  affects social welfare through its effect on  $w^{SP}$ .

To see formally the relationship between  $SW$  and  $\phi$ , differentiate (11) with respect to  $\phi$  to obtain

$$\frac{\partial SW}{\partial \phi} = -\frac{1+s}{2(1-\phi)}w^{SP} + \frac{\psi^{SP}}{(1-2\alpha)(1-\phi)}K^{SP}. \quad (12)$$

Differentiating again with respect to  $\phi$  reveals the second order condition:

$$\frac{\partial^2 SW}{\partial \phi^2} = \frac{2(1-\alpha)\psi^{SP}}{(1-2\alpha)^2(1-\phi)^2}K^{SP} \quad (13)$$

From this, we see that  $\partial^2 SW/\partial \phi^2 > 0$  under our restriction that  $\alpha \in (0, 1/2)$ , and so therefore  $SW$  is strictly convex.

We can now show that  $SW$  may be either monotonically increasing or monotonically decreasing in  $\phi$ , depending on the value of  $w^* + \gamma$ . To see this, note first from (7) that as we make  $w^* + \gamma$  small but still positive,  $w^{SP}$  and hence the first term of (12) becomes small while remaining negative. Next note that  $1/w^{SP}$  becomes large and hence, by (8), so does  $K^{SP}$ , and therefore so does the second term of (12).

Hence,  $\partial SW/\partial \phi$  becomes positive and increasingly large as we make  $w^* + \gamma$  smaller, and increasingly negative as we make  $w^* + \gamma$  larger. This stands to reason since  $l_I^{SP}$ ,  $K^{SP}$ ,  $\pi^{SP}$ ,  $\Pi^{SP}$ , and  $(1 - \phi)(w^{SP} - w^* - \gamma)l_I^{SP}$  are all decreasing in  $w^* + \gamma$ , and so an increase in  $\phi$  has a smaller positive impact on  $SW$  when these elements are small, while  $w^{SP}$  is increasing in  $w^* + \gamma$ , and so an increase

<sup>19</sup>The steps to this derivation are shown in Appendix A.1.



in  $\phi$  has a larger negative impact on  $SW$  when this is large.

Moreover, for given parameter values, there exists a unique finite positive value of  $w^* + \gamma$  at which  $\partial SW/\partial\phi = 0$ . We will denote this value by  $\varpi^{SP}(\phi)$ , to emphasize that it is a function of  $\phi$ .<sup>20</sup> A little further work reveals that  $\varpi^{SP}(\phi)$  is monotonically increasing in  $\phi$ , so it obtains its minimum value at  $\phi = 0$  and its maximum value at  $\phi = \bar{\phi}$ . We will denote  $\varpi^{SP}(0)$  by  $\underline{\varpi}^{SP}$  and  $\varpi^{SP}(\bar{\phi})$  by  $\overline{\varpi}^{SP}$ . Thus, for  $w^* + \gamma < \underline{\varpi}^{SP}$  we have that  $\partial SW/\partial\phi > 0$  for all  $\phi \in [0, \bar{\phi}]$ , and for  $w^* + \gamma > \overline{\varpi}^{SP}$  we have that  $\partial SW/\partial\phi < 0$  for all  $\phi \in [0, \bar{\phi}]$ .

As a result of this monotonicity, for  $w^* + \gamma < \underline{\varpi}^{SP}$ , the solution to the planner's problem requires the planner to set the highest possible naturalization rate,  $\bar{\phi}$ , while for  $w^* + \gamma > \overline{\varpi}^{SP}$ , the solution to the social planner's problem calls for no naturalization, with  $\phi = 0$ .

We can now summarize this discussion as follows.

**Proposition 1:** *The social planner's solution exhibits the following features.*

(i) *The immigration quota, level of capital, profit, net income of capital owners, and quota rent, are all increasing in the naturalization rate, while the domestic wage is decreasing in the naturalization rate.*

(ii) *Social welfare,  $SW$ , is strictly convex in the naturalization rate,  $\phi$ , and monotonically increasing (decreasing) in  $\phi$  for immigrant wage  $w^* + \gamma < \underline{\varpi}^{SP}$  ( $w^* + \gamma > \overline{\varpi}^{SP}$ ).*

(iii) *For  $w^* + \gamma < \underline{\varpi}^{SP}$ , the solution to the planner's problem involves setting the maximum possible naturalization rate,  $\phi = \bar{\phi}$ , while for  $w^* + \gamma > \overline{\varpi}^{SP}$ , this involves having no naturalization at all,  $\phi = 0$ .*

Given the dependency of each of the elements of  $SW$  on  $\phi$ , this result may not seem that surprising in general terms. It says that naturalization leads to a reduction in the domestic wage, and if that has a large enough negative effect on workers' wages relative to the income of capital owners and government revenue, then it can lead to a decline in social welfare.

What makes the result surprising is that we are able to find regions of the parameter space for  $w^* + \gamma$  where the monotonicity of  $SW$  in  $\phi$  is so clear cut, given that each element of  $SW$  depends on  $\phi$  in a non-linear way. This is highlighted in part (ii), which shows that the dependency of  $SW$  on  $\phi$  is determined by whether or not immigrant wages are above  $\overline{\varpi}^{SP}$  or below  $\underline{\varpi}^{SP}$ .

<sup>20</sup>The foregoing discussion is an application of the intermediate value theorem to prove that a unique solution for  $\varpi^{SP}$  exists and is finite. Because of the exponents in (12), there is no clean analytical solution for  $\varpi^{SP}$ .

Thus we have a simple characterization of how  $SW$  depends on  $\phi$ . Focusing on the region above  $\overline{\varpi}^{SP}$  and below  $\underline{\varpi}^{SP}$  rules out, in the simplest way, the possibility that a change in  $\phi$  moves the economy from a region where  $\partial SW/\partial\phi > 0$  to a region where  $\partial SW/\partial\phi < 0$ , or vice versa. Allowing for this possibility would introduce a longer taxonomy of possible outcomes but would not add insight. As we discuss below, we can induce and examine a change in the sign of  $\partial SW/\partial\phi$  in our model by changing  $\gamma$ .

Part (iii) focuses attention on the fact that when  $w^* + \gamma < \underline{\varpi}^{SP}$ , it is socially optimal for the planner to set the naturalization rate at the constrained rate of  $\phi = \bar{\phi}$ , while for  $w^* + \gamma > \overline{\varpi}^{SP}$  it is socially optimal not to provide any naturalization to immigrants at all. In one sense these corner-solution results might be regarded as crude because they imply solutions that are determined by constraints that are outside the scope of the model, especially in the case of  $\bar{\phi}$ . However, this feature will be useful in subsequent analysis because it will enable us to identify a situation where a dictatorship benefits from an absence of naturalization, even though this is damaging for social welfare.

Proposition 1 formalizes the fundamental difference between the institutional function of naturalization as a commitment device, and the way that a trade agreement functions as a commitment device. Both potentially serve to tie the hands of governments to greater openness. We know that, absent terms-of-trade effects, any multilateral trade agreement will increase social welfare because it eliminates distortions that arise from protectionist trade policy. And since we tend to think of immigration and trade liberalization as substitutes, it seems natural to think that a commitment to immigration through naturalization should have a similar welfare-enhancing effect. But Proposition 1 shows that is not necessarily the case.

There are two features of our model that may lead an increase in the naturalization rate to reduce social welfare. One is that immigration here leads to an increase in labor supply, which in turn leads to a reduction in the domestic wage. So immigration through naturalization here has an ‘immiserizing growth effect’ that may lead to a fall in overall national income through an increase in factor supply. The second is that, since  $\gamma > 0$ , there is a distortion present in our model in the form of quota rent going to immigrants that remains in place through the process of naturalization. This puts us in a second-best world whereby policy liberalization can have a deleterious effect on social welfare. Indeed, for initial values of  $w^* + \gamma > \overline{\varpi}^{SP}$ , where  $w^* < \underline{\varpi}^{SP}$ , a reduction of  $\gamma$  can move us from a world in which an increase in  $\phi$  reduces social welfare to one where an increase in  $\phi$  increases social welfare.

## 4 Government with ‘No Lobbying’

We proceed to consider how the government sets immigration policy to maximize its objective function, (5), starting with the benchmark case where it is constrained not to receive financial contributions from the lobby. Like the planner, the government sets immigration policy to maximize its objective function. Unlike the planner, the government cannot set  $\phi$  but instead takes it as given. Here, capital owners determine the level of capital. We are interested in whether the government could gain by having its hands tied to naturalization.

For concreteness, let us say that a country’s ‘founding forefathers’ were able to ‘enshrine’ its level of  $\phi$  in the country’s constitution. This then constrains the government to adopt the enshrined level of  $\phi$ . Therefore, if a government is constrained by naturalization then it must adopt  $\bar{\phi} > 0$ . On the other hand, if the country is not constrained by naturalization then this is because the founding forefathers have enshrined  $\phi = 0$ , and not constrained its government to a positive naturalization rate.

The sequence of events is as follows. In Stage 1, capital owners determine the level of capital to maximize their profit. Then, in Stage 2, for a given quantity of capital the government determines the immigration quota to maximize its payoff. Then, conditional on the level of capital and the immigration quota, markets clear, prices are determined, and consumption takes place. If the government’s hands are tied to a given naturalization rate, either  $\phi = 0$  or  $\phi = \bar{\phi}$ , then this takes place prior to Stage 1. We use backward induction to analyze this game, whether it has two or three stages.

Under the ‘no lobbying’ constraint, the government payoff function becomes:

$$GW(l_I : c(l_I) = 0) = zs\Pi(l_I) + (1 + \phi l_I)w(l_I) + (1 - \phi)(w(l_I) - w^* - \gamma)l_I + a0. \quad (14)$$

This has the same basic form as (5), with the final term  $a0$  reflecting the fact that under government with no lobbying (GNL), contributions are constrained to be zero:  $c(l_I) = 0$ .

### 4.1 Stage 2

Using superscript ‘NL’ to denote variables under GNL, and using the solutions for  $r$  and  $w$ , plus the expression for  $\pi$ , (1), we obtain the following expression for the government-payoff-maximizing

immigration quota under GNL,  $l_I^{NL}$ :

$$l_I^{NL} = \left( \frac{1}{2w^{NL}} \right)^2 K^{2\alpha} - 1, \quad (15)$$

where  $w^{NL} = 2(1 - \phi)(w^* + \gamma)/(1 + zs)$  and  $K$  is taken as given. Since the form of  $GW(l_I : c(l_I) = 0)$  is the same as  $SW(l_I)$  except for the additional weight  $z$  placed on  $\Pi(l_I)$ , the solution for  $l_I^{NL}$  given by (15) has the same basic form as the solution  $l_I^{SP}$  given by (6). The difference takes into account the weight  $z$  and the fact that the level of  $K$  may be different. Thus, the basic tensions reflected in the determination of the immigration quota under GNL are the same as those under the social planner, except that the greater the weight  $z > 1$  that the government places on the welfare of capital owners, the higher is the immigration quota,  $l_I^{NL}$ .

The domestic wage under GNL has the same basic form as under the planner's solution, but any additional weight  $z$  that the government gives to capital owners' welfare puts downward pressure on the domestic wage. To ensure that  $w^{NL} > w^* + \gamma$  if  $z > 1$ , we will modify our condition on  $s$  to  $s < (1 - 2\phi)/z$ . Holding  $K$  fixed and setting  $z = 1$ , the solutions under GNL for profit,  $\pi^{NL}$ , and net income of capital owners,  $\Pi^{NL}$ , are the same as under the planner's solution.

## 4.2 Stage 1

Anticipating the government's choice of  $l_I^{NL}$  in Stage 2, in Stage 1 capital owners choose  $K^{NL}$  to maximize their net income,  $\Pi$ , given by (3). This yields:

$$K^{NL} = \left( \frac{\alpha}{2} \frac{1}{w^{NL}} \right)^{\frac{1}{1-2\alpha}}. \quad (16)$$

This solution makes intuitive sense: the greater is the importance of capital in production,  $\alpha$ , the greater is the incentive to invest; the lower is the domestic wage, and underpinning that the immigrant wage  $w^* + \gamma$ , the greater is the incentive to invest in capital.

Let us now compare  $l^{NL}$  to  $l^{SP}$ ,  $K^{NL}$  to  $K^{SP}$ ,  $\pi^{NL}$  to  $\pi^{SP}$ , and  $\Pi^{NL}$  to  $\Pi^{SP}$ , taking the naturalization rate,  $\phi$ , as given. To do this, we will assume  $z = 1$ , making  $SW(l_I) = GW(l_I : c(l_I) = 0)$  and hence making all functions directly comparable to one another. Then the solutions for  $l^{NL}$  and  $l^{SP}$  are the same for given  $K$ . By comparison of (8) and (16),  $K^{SP}$  is larger than  $K^{NL}$ , for two reasons. First, while the planner values the effect of investment on the quota rent, capital owners only value the role that capital plays in generating profit for themselves. For given quota rent per immigrant  $(w - w^* - \gamma)$  the quota is increasing in  $K$ . So the planner has an incentive to increase  $K$  relative

to capital owners because of the effect on quota rent. Entrepreneurs ignore this connection because quota rents have no bearing on their net income.

Second, while under the social planner there is obviously no hold-up problem, under GNL capital owners anticipate that the government will hold them up by tightening the immigration quota after capital is sunk, and reduce investment accordingly. Given  $K^{NL} < K^{SP}$ ,  $z = 1$ , values of all other parameters, and any given naturalization rate  $\phi \in [0, \bar{\phi}]$ , by (6) and (15),  $l^{NL} < l^{SP}$ . In addition, using  $K^{NL}$  instead of  $K^{SP}$  in (9) and (10),  $\pi^{NL} < \pi^{SP}$ , and  $\Pi^{NL} < \Pi^{SP}$ .

From this we can see that the government's incentive to hold up capital owners, and capital owners' anticipation of this, leads to under-investment in capital, and this in turn manifests in all outcomes being sub-optimal except for wages. For  $z = 1$ ,  $w^{NL} = w^{SP}$  because, even though immigration policy is distorted by the government's incentive to hold up capital owners, capital owners anticipate this and scale back investment just to the point where the return to labor is the same as under the planner's problem.

### 4.3 Naturalization

The naturalization rate has the same qualitative effect under GNL as it does under the planner's solution, but the way it works is different. Naturalization mitigates the hold-up problem by limiting the government's incentive to tighten  $l_I$  after capital owners have sunk their investment, thereby reducing the domestic wage, increasing profit and net income of capital owners for given capital. Anticipating this, capital owners increase capital formation in Stage 1. Although  $l_I^{NL}$  increases, the total quota rent collected by the government  $(1 - \phi)(w^{NL} - w^* - \gamma)l_I^{NL}$  may be decreased because  $w^{NL}$  is lowered by naturalization. So we can see that increased investment due to a higher naturalization rate comes at the cost of a lower domestic wage, and possibly lower government revenue.

We can now analyze the overall effect of naturalization on the government's payoff. To do so, we use the solutions for  $w^{NL}$ ,  $l_I^{NL}$ ,  $K^{NL}$ ,  $\pi^{NL}$ , and  $\Pi^{NL}$  in (14) to obtain a reduced-form expression for the government payoff function:

$$GW(l_I : c(l_I) = 0) = \frac{1 + zs}{2} w^{NL} + \psi^{NL} K^{NL}, \quad (17)$$

where

$$\psi^{NL} = \left( \frac{1 + zs(1 - 4\alpha)}{4\alpha} \right) > 0. \quad (18)$$

By writing  $GW(l_I : c(l_I) = 0)$  in (17) in the same way as we wrote  $SW$  in (11), we once again highlight the tension between the element that is increasing in the domestic wage, in this case  $w^{NL}$ , and the element that is decreasing in  $w^{NL}$  via its effect on  $K^{NL}$ : recall equation (16). Set  $z = 1$ , so that  $w^{NL} = w^{SW}$ . Then we can see by comparison of (17) to (11) that the basic form of  $GW(l_I : c(l_I) = 0)$  is the same as  $SW$ , except for the fact that  $\psi^{NL}$  is different from  $\psi^{SP}$  and obviously  $K^{NL} < K^{SP}$ . Consequently, like for the planner's problem, the effect of an increase in  $\phi$  on the government's payoff depends on  $w^{NL}$  and, underpinning that, the immigrant wage  $w^* + \gamma$ .

Using (17) to differentiate  $GW(c(l_I) = 0)$  with respect to  $\phi$ , and expressing the result in the same way as we did for  $\partial SW / \partial \phi$  in (12), we obtain

$$\frac{\partial GW(l_I : c(l_I) = 0)}{\partial \phi} = -\frac{1 + zs}{2(1 - \phi)} w^{NL} + \frac{\psi^{NL}}{(1 - 2\alpha)(1 - \phi)} K^{NL} \quad (19)$$

Differentiating again with respect to  $\phi$  reveals  $\partial^2 GW(l_I : c(l_I) = 0) / \partial \phi^2 > 0$  under our restriction that  $\alpha \in (0, 1/2)$ , and so therefore  $GW(c(l_I) = 0)$  is strictly convex in  $\phi$ .

Like for  $SW$ ,  $GW$  is either monotonically increasing, or monotonically decreasing, in  $\phi$ . This again depends on  $w^* + \gamma$ . If  $w^* + \gamma$  is small but still positive,  $w^{NL}$  and hence the first term of (19) is negative and small in absolute value, while  $1/w^{NL}$ , and hence  $K^{NL}$ , and hence the second term of (19) is positive and large. Thus, all else equal,  $\partial GW(c(l_I) = 0) / \partial \phi$  is increasingly positive (negative) as we make  $w^* + \gamma$  smaller (larger).

The effects on the underlying variables  $l_I^{NL}$ ,  $K^{NL}$ ,  $\pi^{NL}$ ,  $\Pi^{NL}$ , and  $(1 - \phi)(w^{NL} - w^* - \gamma)l_I^{NL}$  are all qualitatively the same as they are in the planner's solution. Moreover, for given parameter values, there exists a unique finite positive value of  $w^* + \gamma$  at which  $\partial GW(l_I : c(l_I) = 0) / \partial \phi = 0$ . We will denote this value of  $w^* + \gamma$  by  $\varpi^{NL}(\phi)$ , in parallel to our derivation of  $\varpi^{SW}(\phi)$  above. Accordingly, we will denote  $\varpi^{NL}(0)$  by  $\underline{\varpi}^{NL}$  and  $\varpi^{NL}(\bar{\phi})$  by  $\overline{\varpi}^{NL}$ . As we can see, in qualitative terms, our characterization of the way that an increase in  $\phi$  affects social welfare carries over to the way that an increase in  $\phi$  affects the government's payoff.

Since the effect of  $\phi$  on  $GW(l_I : c(l_I) = 0)$  is monotonic for  $w^* + \gamma < \underline{\varpi}^{NL}$  and  $w^* + \gamma > \overline{\varpi}^{NL}$ , we can use the above analysis to infer the effect on  $GW(l_I : c(l_I) = 0)$  of adoption of naturalization, which we have formalized by an increase in  $\phi$  from  $\phi = 0$  to  $\phi = \bar{\phi}$ . If  $w^* + \gamma < \underline{\varpi}^{NL}$  then  $\partial GW(c(l_I) = 0) / \partial \phi > 0$  for any  $\phi \in [0, \bar{\phi}]$ , and so  $GW(l_I : c(l_I) = 0)$  must increase for an increase of  $\phi$  from  $\phi = 0$  to  $\phi = \bar{\phi}$ . If on the other hand  $w^* + \gamma > \overline{\varpi}^{NL}$  then  $\partial GW(l_I : c(l_I) = 0) / \partial \phi < 0$  for any  $\phi \in [0, \bar{\phi}]$ , and so  $GW(l_I : c(l_I) = 0)$  must decrease for such an increase in  $\phi$ .

We would like to be able to make comparisons between the planner's solution and the outcome under GNL. To do this in the clearest way possible, it will be convenient to restrict attention to ranges of the parameter space where either  $SW$  and  $GW(l_I : c(l_I) = 0)$  are both increasing in  $\phi$ , or  $SW$  and  $GW(l_I : c(l_I) = 0)$  are both decreasing in  $\phi$ . For this purpose, let  $\underline{\varpi} = \min \{\underline{\varpi}^{SW}, \underline{\varpi}^{NL}\}$ , and  $\bar{\varpi} = \max \{\bar{\varpi}^{SW}, \bar{\varpi}^{NL}\}$ . Then we can say for sure that if  $w^* + \gamma < \underline{\varpi}$ , both  $\partial SW / \partial \phi > 0$  and  $\partial GW(l_I : c(l_I) = 0) / \partial \phi > 0$  for all  $\phi \in [0, \bar{\phi}]$ . And if  $w^* + \gamma > \bar{\varpi}$ , then both  $\partial SW / \partial \phi < 0$  and  $\partial GW(l_I : c(l_I) = 0) / \partial \phi < 0$  for all  $\phi \in [0, \bar{\phi}]$ . Using  $\underline{\varpi}$  and  $\bar{\varpi}$ , we can state the following result that compares the outcome under GNL with the planner's solution.

**Proposition 2:** *Fix  $z = 1$ . GNL exhibits the following features.*

(i) *The immigration quota, level of capital, profit, net income of capital owners, and quota rent, are all increasing in the naturalization rate,  $\phi$ , while the domestic wage is decreasing in the naturalization rate.*

(ii) *The government's payoff,  $GW(l_I : c(l_I) = 0)$ , is strictly convex in the naturalization rate,  $\phi$ , and monotonically increasing (decreasing) in  $\phi$  for immigrant wage  $w^* + \gamma < \underline{\varpi}^{NL}$  ( $w^* + \gamma > \bar{\varpi}^{NL}$ ).*

(iii) *If  $w^* + \gamma < \underline{\varpi}$  then, all else equal, the government's payoff is maximized if its hands are tied to naturalization,  $\phi = \bar{\phi}$ , and this also maximizes social welfare. If  $w^* + \gamma > \bar{\varpi}$  then, all else equal, the government's payoff is maximized if its hands are not tied to naturalization,  $\phi = 0$ , and this also maximizes social welfare.*

Parts (i) and (ii) of Proposition 2 directly parallel parts (i) and (ii) of Proposition 1. This is helpful because it illustrates the commonalities between the social planner's problem and that of the government.

Part (iii) is similar to Part (iii) of Proposition 1: it shows that, like for social welfare, when the immigrant wage is sufficiently low, the government's payoff will be maximized if it is constrained by naturalization, and that this will maximize social welfare as well. On the other hand, when the immigrant wage is sufficiently high, the government's payoff will be maximized if its hands are not tied to naturalization, and this will maximize social welfare as well.

The difference in the result reflects our assumption that, differently from the social planner, the government does not get to choose the naturalization rate because it is set exogenously, and by assumption does not even get to choose whether or not to adopt naturalization.

We have focused in the above characterization of equilibrium under GNL on the overall level

of the government's payoff and the social welfare of this outcome. In the next result, we consider in greater detail specific economic outcomes under GNL and how these compare to those under the planner's solution.

**Proposition 3:** *Fix  $z = 1$ , and assume either that  $w^* + \gamma < \underline{\omega}$ , or that  $w^* + \gamma > \bar{\omega}$ . Under GNL, the level of capital is lower than under the planner's solution because the planner internalizes the effect of investment on quota rents, while under GNL (i) capital owners do not internalize the effect of investment on quota rents, and (ii) capital owners anticipate being held up by the government when it sets the immigration quota. It follows that the immigration quota, profit, and the net income of capital owners are lower under GNL than under the planner's solution, but the domestic wage is the same under both regimes.*

Proposition 3 says that the level of capital, immigration, profits, and net income of capital owners are inefficiently low under GNL. This holds regardless of whether  $w^* + \gamma$  is sufficiently low, or sufficiently high, to ensure that both the social welfare function and the government's payoff function are monotonically increasing, or monotonically decreasing, in  $\phi$ .

#### 4.4 Dictatorship and Democracy with 'No Lobbying'

Until this point in our analysis we have imposed the assumption that  $z = 1$ , in order to focus on the commonalities and differences between the planner's problem and GNL. We now relax this assumption to shift focus onto how outcomes may differ according to the form of government: democracy or dictatorship. Accordingly, we now relax our assumption that  $z = 1$ , and allow for the possibility that  $z > 1$ . In line with the discussion above, we further assume  $s < (1 - 2\phi) / z$ .

This allowance for  $z > 1$  enables us to compare economic outcomes between democracy and dictatorship. By inspection of (15), we see that, taking  $K$  as given, the immigration quota,  $l_I^{NL}$ , is increasing in  $z$ . And we see from (16) that the level of capital,  $K^{NL}$ , is increasing in  $z$ . This says that, all else equal, both capital and the immigration quota will be higher under dictatorship than under democracy. On the other hand, the domestic wage  $w^{NL}$  is decreasing in  $z$ , so the domestic wage will be lower under dictatorship than under democracy. So there is a clear conflict of interest between capital owners and workers over the form of government. Capital owners will prefer dictatorship while workers will prefer democracy.

The natural question that follows is whether naturalization is more likely to lead to an increase



in the government payoff under democracy or dictatorship. We can consider this by examining the effect of an increase in  $z$  on  $\partial GW(l_I : c(l_I) = 0) / \partial \phi$ . Differentiating (19) with respect to  $z$ , we obtain

$$\frac{\partial^2 GW(l_I : c(l_I) = 0)}{\partial \phi \partial z} = \frac{1}{(1 - 2\alpha)(1 - \phi)} \left( \frac{s}{(1 + zs)(1 - 2\alpha)} \psi^{NL} + \frac{\partial \psi^{NL}}{\partial z} \right) K^{NL} > 0$$

The fact that  $\partial \psi^{NL} / \partial z > 0$  follows by inspection of (18). The fact that  $\partial GW(l_I : c(l_I) = 0) / \partial \phi$  is continuous and monotonically increasing in  $z$  means that  $\partial GW / \partial \phi$  is increasing in  $z$  not just for small increases in  $z$  but for a discrete increase in  $z$  to  $z' > z$ . Note that it is always possible to choose a value of  $s$  sufficiently small that  $s < (1 - 2\phi) / z$  and  $s < (1 - 2\phi) / z'$ . We will assume that  $s$  is chosen to satisfy this restriction throughout the analysis. We can now use our solution for  $\partial^2 GW(l_I : c(l_I) = 0) / \partial \phi \partial z$  to obtain the following result.

**Lemma 1:** *Assume an initial set of parameters, including  $z \geq 1$ , for which*

*$\partial GW(l_I : c(l_I) = 0) / \partial \phi < 0$ . Then there exists a value of  $z' > z$  sufficiently large that, all else equal,  $\partial GW(l_I : c(l_I) = 0) / \partial \phi > 0$ .*

Lemma 1 says that if we start from a set of parameters for which the government's payoff is decreasing in the naturalization rate,  $\partial GW(l_I : c(l_I) = 0) / \partial \phi < 0$ , there always exists a value of  $z$  sufficiently large that, all else equal, the government's payoff becomes increasing in the naturalization rate:  $\partial GW(l_I : c(l_I) = 0) / \partial \phi > 0$ .

To see what we learn from Lemma 1, think of a democracy for which  $z = 1$ , and whose parameters give rise to  $\partial GW(l_I : c(l_I) = 0) / \partial \phi < 0$ . Lemma 1 then tells us that we could have a dictatorship that is otherwise identical to the democracy except the weight that its government places on the welfare of capital owners,  $z' > 1$ , is sufficiently large that  $\partial GW(l_I : c(l_I) = 0) / \partial \phi > 0$ . This in turn tells us that under GNL, it is actually dictatorships whose government payoffs have a greater tendency than democracies to be increasing in the naturalization rate. We will now summarize this discussion, and then use it to provide an interpretation of the literature.

**Proposition 4:** *Assume that under GNL, democracy is represented by  $z = 1$ , and dictatorship is represented by  $z' > 1$ . Then, all else equal, both capital and the immigration quota will be higher under dictatorship than under democracy. In addition, there is a greater tendency for  $GW(l_I : c(l_I) = 0)$  to be maximized at  $\phi = 0$  under democracy, and a greater tendency for  $GW(l_I : c(l_I) = 0)$  to be maximized at  $\phi = \bar{\phi}$  under dictatorship.*

This result discusses two tendencies. First, the immigration quota is higher under dictatorship than

under democracy. Second, the government's payoff is more likely to be maximized under naturalization for a dictatorship than for a democracy. If this second tendency is enshrined in the constitution by a country's founding forefathers, we should actually expect to see a greater tendency for democracies not to naturalize, and naturalization to be enshrined in the constitutions of dictatorships. The fact that we observe a pattern in the data that contradicts this second tendency is precisely the Koopmans-Michalowski paradox that we discussed in the Introduction. Our analysis to this point provides the first formalization of this paradox.

The conclusion we draw from our analysis so far is that it shows an important feature is missing from our model as specified. We argue that the missing feature is that capital owners lobby the government over immigration policy. In the next section we show, by introducing lobbying, that the model's prediction is modified such that while dictatorships are more open when it comes to immigration policy, democracies have a greater tendency to extend naturalization rights to immigrants.

## 5 Government with Lobbying

In our analysis of government with lobbying (GWL), we relax our assumption that there is no lobbying, by removing the constraint that the government cannot accept financial contributions. So now the government sets  $l_I$  to maximize  $GW(l_I : c(l_I) \geq 0)$ , or  $GW(l_I)$  for short.

The sequence of events is the same as for GNL. The key difference is that for GWL, in Stage 2, the capital owners can lobby the government to increase the immigration quota. Under GWL, the government weighs the returns to domestic workers against capital owners as it did under GNL, but now also weighs these against the lobby contribution it receives from capital owners via the lobby. Thus, in Stage 1 capital owners choose the amount of capital to invest, while in Stage 2 the lobby and the government bargain over setting the immigration quota and the size of the lobby's contribution.

Following Maggi and Rodríguez-Clare (1998), bargaining takes Nash form. This bargaining process leads to the determination of the immigration quota and contributions. If the government is constrained by naturalization then this constraint is imposed prior to Stage 1. Like for GNL, we use the method of backward induction to solve for the equilibrium under GWL.

## 5.1 Stage 2

We will use the superscript 'NB' to denote solutions for corresponding variables under lobbying, arising through a Nash Bargain. Following the standard Nash Bargaining approach, the immigration quota  $l_I^{NB}$  maximizes the joint surplus of the government and the lobby,  $JS$ , represented as:

$$JS = GW(l_I) + a(zs(\pi(l_I) + r(l_I)K - K) - c(l_I)l_I). \quad (20)$$

Under GWL, using (1) and (5) in (20), plus the solutions for  $r$  and  $w$ , the level of immigration that maximizes (20) is:

$$l_I^{NB} = \left( \frac{1}{2w^{NB}} \right)^2 K^{2\alpha} - 1. \quad (21)$$

where

$$w^{NB} = \frac{2(1 - \phi)(w^* + \gamma)}{(1 + zs(1 + a))} \quad (22)$$

and  $K$  is taken as given. By inspection, we can see that the solution for  $l_I^{NB}$  given by (21) has the same form as the solution for  $l_I^{NL}$  given by (15), except that  $l_I^{NB}$  is increasing in  $a$  through the effect of  $a$  on  $w^{NB}$ . That is, all else equal, the quota under GWL is higher than under GNL and this depends on  $a$ . This reflects the fact that capital owners can now lobby to increase the immigration quota. The corresponding domestic wage under GWL,  $w^{NB}$ , is lower than  $w^{NL}$  because of the larger number of immigrants that arise from  $l^{NB} > l^{NL}$ .

Under Nash Bargaining, the government and the lobby share the surplus obtained from the increased immigration according to their bargaining powers. Denote by  $B$  the 'Nash product' of the surplus to the government and capital owners that arises from GWL, where their bargaining powers are denoted by  $\sigma$  and  $1 - \sigma$  respectively, and take capital as given at  $K$ . Then we have:

$$B = (GW(l_I^{NL} : c(l_I) = 0) - GW(l_I^{NB}))^\sigma \times (\pi(l_I^{NB}) + r(l_I^{NB})K - K - c(l_I^{NB})l_I^{NB} - \pi(l_I^{NL}) - r(l_I^{NL})K + K)^{1-\sigma}.$$

Following the standard Nash Bargaining procedure, contributions,  $c(l_I^{NB})$ , are chosen to maximize  $B$ . This yields a total contribution schedule:

$$c(l_I^{NB})l_I^{NB} = \frac{(1 - \sigma) (GW(l_I^{NL} : c(l_I) = 0) - GW(l_I^{NB}))}{a} + \sigma (\pi(l_I^{NB}) + r(l_I^{NB})K - \pi(l_I^{NL}) - r(l_I^{NL})K).$$

Using (5), (15) and (21), the reduced-form expression for optimal total contributions is as follows:

$$c(l_I^{NB})l_I^{NB} = \frac{a z s (2\sigma + z s (1 - \sigma))}{16(1 - \phi)(w^* + \gamma)} K^{2\alpha}. \quad (23)$$

The implications of the solutions for  $l_I^{NB}$  and  $c(l_I^{NB})$  can be summarized as follows.

**Lemma 2:** *In Stage 2 of GWL, the level of lobby contributions made by capital owners,  $c(l_I^{NB})$ , and hence the immigration quota,  $l_I^{NB}$ , are increasing in the degree of responsiveness of the government to lobby contributions,  $a$ , the naturalization rate,  $\phi$ , the weight that the government puts on the income of capital owners,  $z$ , and on the level of capital investment, but decreasing in the immigrant wage,  $w^* + \gamma$ . Also, the greater the bargaining power of the government,  $\sigma$ , the larger the contributions it will be able to extract from the lobby.*

**Proof.** For an elaboration of details not covered in the above discussion, see Appendix A.2 ■

The most important aspect of the equilibrium in Stage 2, revealed by Lemma 2, concerns the effect of the government's bargaining power on the outcome. When the government's bargaining power is at its maximum, i.e.  $\sigma = 1$ , the contributions that capital owners will have to pay is exactly equal to the difference in their surplus between GWL and GNL. Thus, the government will be able to extract all the additional surplus made by capital owners under GWL. On the other hand, if capital owners' bargaining power is at its maximum, i.e.  $\sigma = 0$ , the contribution that the government extracts will be just sufficient to compensate it for the difference in social welfare between GWL and GNL, with all the remaining surplus going to capital owners.

## 5.2 Stage 1

In Stage 1, anticipating the bargaining outcome of Stage 2 over the immigration quota and contributions, capital owners determine the level of capital. Modifying (3) to account for the fact that, under GWL, capital owners must also factor in the cost of contributions,

$$\Pi(l_I^{NB}) = \pi(l_I^{NB}) + r(l_I^{NB})K - K - c(l_I^{NB})l_I^{NB}.$$

Capital owners choose  $K$  to maximize this function. The difference between this and the problem they solve under GNL is that here capital owners also account for the contributions that they pay to the government and how the resulting change in the immigration quota affects their incentive to invest in capital. To obtain a reduced-form expression for  $\Pi(l_I^{NB})$ , substitute into the above for  $l_I^{NB}$  using (21) and (22), and use (23) to substitute for  $c(l_I^{NB})l_I^{NB}$ .

From the reduced-form expression for  $\Pi(l_I^{NB})$ , we obtain the following net-income maximizing solution for capital under GWL,  $K^{NB}$ :

$$K^{NB} = \left( \frac{\alpha}{2} \left( \frac{1}{w^{NB}} - \frac{zs(a\sigma + azs(1-\sigma)/2)}{2(1-\phi)(w^* + \gamma)} \right) \right)^{\frac{1}{1-2\alpha}}. \quad (24)$$

It is clear by inspection that  $K^{NB}$  is decreasing in  $\sigma$ .<sup>21</sup> This is because capital owners anticipate having to make larger contributions to relax the immigration quota when the government is stronger and at the margin this reduces the incentive to invest.

A little further work establishes that  $K^{NB} = K^{NL}$  when  $K^{NB}$  achieves a minimum at  $\sigma = 1$ . At the other end of the spectrum, when  $\sigma = 0$ ,

$$K^{NB} = \left( \frac{\alpha}{2} \left( \frac{1}{w^{NB}} - \frac{az^2s^2}{4(1-\phi)(w^* + \gamma)} \right) \right)^{\frac{1}{1-2\alpha}}.$$

From this solution, and using (22) to substitute for  $w^{NB}$ ,  $K^{NB}$  is increasing in  $a$ . This stands to reason since, if the government cares enough about lobby revenues, it can be induced to relax the immigration quota in Stage 2 and, anticipating this, capital owners will be prepared to invest more in capital in Stage 1. To compare this outcome with the efficient level, set  $z = 1$ . Then it is straight forward to verify that  $K^{NB} > K^{SP}$  for  $a$  sufficiently large. So the level of capital can be inefficiently large in equilibrium under GWL.

Allowing  $z$  to vary, and using (22) in (24), we find that  $K^{NB}$  is increasing in  $z$ . This, too, stands to reason. The more that the government cares about capital owners, the more it will relax the immigration quota, thus raising the profit for capital owners and their incentive to invest.

Equation (24) holds the key to understanding how the outcome for investment under GWL differs from GNL. When government bargaining power is at  $\sigma = 1$ , so that the government can extract all the surplus from the bargain, capital owners have no incentive to invest above the level that they would under GNL where they cannot lobby: hence  $K^{NB} = K^{NL}$ . As government bargaining power is weakened, reflected by a reduction in  $\sigma$ , capital owners are able to reap more of the surplus created by the bargain and so have an incentive to invest more. If the government cares enough about lobby contributions, the incentive to invest can even become inefficiently high. But note that, even if  $\sigma$  and  $a$  are such that  $K^{NB} \geq K^{SP}$ , the outcome will be sub-optimal from a social welfare standpoint because the wage is always inefficiently low under GWL.

<sup>21</sup>Recall the condition on  $s$  that we imposed earlier,  $s < (1 - 2\phi)/z$ , which here ensures  $2a\sigma + azs(1 - \sigma) > 0$ , and so (24) is decreasing in  $\sigma$ .

We can also see that, like under GNL, capital formation is increasing in naturalization,  $\phi$ . As under GNL, capital owners are guaranteed the presence in the country of naturalized immigrants, since they are placed beyond the bargain over immigration policy, and this increases the incentive to invest in capital. Like under GNL, naturalization provides a way to mitigate the hold-up problem under GWL.

We now have a complete characterization of equilibrium under GWL, taking the naturalization rate  $\phi$  as given.

**Proposition 5:** *For  $\sigma = 1$ , capital investment is equal under GWL to GNL. As government bargaining power  $\sigma$  is reduced, investment under GWL increases relative to GNL. This is because capital owners gain an increased share through lobbying of the increased surplus generated by investment in exchange for an increase in the immigration quota, and this increases their incentive to invest. The larger is  $a$ , the larger the increase in investment through a reduction in  $\sigma$ . In addition, the greater is the naturalization rate,  $\phi \in [0, \bar{\phi}]$ , the greater will be capital investment and the lower will be the domestic wage. The domestic wage is lower under GWL than under GNL, but it is unaffected by government bargaining power.*

**Proof.** For an elaboration of details not covered in the above discussion, see Appendix A.2. ■

The first part of Proposition 5 can be interpreted in terms of the hold-up problem. Capital owners and the government enter into a bargain through lobbying that yields both of them a higher return than under GNL, because the bargain offers them the opportunity to mitigate the hold-up problem. However, capital owners know that once they have made an investment in Stage 1, they will have to share the gross returns from this investment with the government in Stage 2. Consequently, the higher is the government's bargaining power the less capital owners will invest, anticipating that a greater portion of their returns post-negotiation will accrue to the government. Of course, the more responsive the government is to lobby contributions, the more they will be prepared to relax the immigration quota in response to a given level of contributions and hence the greater the incentive for capital owners to invest.

Turn now to the second part of Proposition 5. The higher the naturalization rate, for a given stock of capital, the higher the immigration quota since the government has an incentive to capture more quota rent from those immigrants who are not naturalized. The higher the immigration quota, the greater the downward pressure on the domestic wage. At the same time, the higher the naturalization

rate the more the hold-up problem is mitigated, as we saw under GNL. This implies that capital investment is increased, as a result of which there is upward pressure on the domestic wage. This increase in capital leads to a further increase in the immigration quota, which puts further downward pressure on the domestic wage. Although there are two opposing effects on the domestic wage, the downward pressure dominates. From the expression for the domestic wage,  $w^{NB}$ , we can confirm that for any given parameters, the wage is decreasing in  $\phi$ .

### 5.3 Naturalization

Following the same steps in our analysis under GWL as we did for GNL, we will now examine whether there are any circumstances under which the government's payoff will be increased by having its hands tied to a positive naturalization rate: moving from  $\phi = 0$  to  $\phi > 0$ .

To consider this, use (2), (3), (21), (23) and (24) to write the government's payoff function under GWL as

$$GW(l_I) = \frac{1 + zs(1 + a)}{2} w^{NB} + \psi^{NB} K^{NB}, \quad (25)$$

where

$$\psi^{NB} = \frac{1 + zs(2 - 4\alpha + zs(1 - 2(2 + a(2 - zs))\alpha) + a(2 - zs)(a + 2zs\alpha)\sigma)}{2\alpha(2 + zs(2 + a(2 - zs)(1 - \sigma)))} \quad (26)$$

Like for  $SW$  and  $GW(c(l_I) = 0)$ , writing  $GW(l_I)$  in the form of (25) highlights the tension between elements that are increasing in the domestic wage, now  $w^{NB}$ , and those that are decreasing in  $w^{NB}$  via their effect on  $K^{NB}$ : see equation (24).

To make  $GW(l_I)$  comparable to  $SW$  and  $GW(c(l_I) = 0)$  it will be convenient to introduce a parameter restriction on  $a$  to ensure that  $\psi^{NB} > 0$ , since  $\psi^{NB}$  is decreasing in  $a$ . The necessary condition is extremely complex. However, since from (26) we can see that  $\psi^{NB}$  is increasing in  $\sigma$ , we can obtain a sufficient condition for  $a$  to ensure that  $\psi^{NB} > 0$  by setting  $\sigma = 0$  and solving for the value of  $a$  at which the numerator of the expression in the second set of brackets of (26) is positive. Denoting this upper bound on  $a$  by  $\bar{a}$ , we have

$$\bar{a} = \frac{(1 + zs)(1 + zs(1 - 4\alpha))}{2z^2s^2(2 - zs)\alpha}$$

It is immediately evident that  $\bar{a} > 0$  under the parameter restrictions we have made so far. And because  $\psi^{NB}$  is increasing in  $\sigma$ , if  $a < \bar{a}$  then  $\psi^{NB} > 0$  for all  $\sigma \in [0, 1]$ . Moreover, it is straight forward to verify that  $\bar{a}$  can be made arbitrarily large by making  $s$  sufficiently small. Note that

assuming  $a < \bar{a}$  does not rule out the possibility that  $K^{NB} > K^{SP}$ . We will assume that  $a < \bar{a}$  throughout the remainder of the analysis.

Taking the derivative of  $GW(l_I)$  with respect to  $\phi$  gives us

$$\frac{\partial GW(l_I)}{\partial \phi} = -\frac{1 + zs(1+a)}{2(1-\phi)}w^{NB} + \frac{\psi^{NB}}{(1-2\alpha)(1-\phi)}K^{NB} \quad (27)$$

This has the same basic form as  $\partial SW/\partial \phi$  and  $\partial GW(l_I : c(l_I) = 0)/\partial \phi$ , except that  $\partial GW(l_I)/\partial \phi$  incorporates terms in  $a$ . In addition, here we require  $a < \bar{a}$  to ensure that  $\psi^{NB} > 0$ , whereas  $\psi^{SW} > 0$  and  $\psi^{NL} > 0$  under existing parameter restrictions. Since  $\partial^2 GW(l_I)/\partial \phi^2$  has the same basic form as  $\partial^2 SW/\partial \phi^2$  shown by (13), whereby  $\psi^{SP} > 0$  guarantees  $\partial^2 SW/\partial \phi^2 > 0$ , we also have that since  $a < \bar{a}$  guarantees  $\psi^{SP} > 0$ , it guarantees  $\partial^2 GW(l_I)/\partial \phi^2 > 0$  and hence the convexity of  $GW(l_I)$  in  $\phi$  as well.

Now, given  $a < \bar{a}$ , like for  $SW$  and  $GW(l_I : c(l_I) = 0)$ ,  $GW(l_I)$  is either monotonically increasing, or monotonically decreasing, in  $\phi$ . This again depends on  $w^* + \gamma$ , which if small means  $w^{NB}$  is small and hence the first term of (19) is negative and small in absolute value, while  $1/w^{NB}$  and hence the second term is positive and large in absolute value. This makes  $\partial GW(l_I)/\partial \phi$  increasingly positive (negative) as we make  $w^* + \gamma$  smaller (larger).

Moreover, like for  $SW$  and  $GW(l_I : c(l_I) = 0)$ , for given parameter values, there exists a unique finite positive value of  $w^* + \gamma$  at which  $\partial GW(l_I)/\partial \phi = 0$ . We will denote this value of  $w^* + \gamma$  by  $\varpi^{NB}(\phi)$ , in parallel to our derivation of  $\varpi^{SW}(\phi)$  and  $\varpi^{NL}(\phi)$  above. Accordingly, we will denote  $\varpi^{NB}(0)$  by  $\underline{\varpi}^{NB}$  and  $\varpi^{NB}(\bar{\phi})$  by  $\overline{\varpi}^{NB}$ . As we can see, in qualitative terms, our characterization of the way that an increase in  $\phi$  affects social welfare carries over to the way that an increase in  $\phi$  affects the government's payoff, not just under 'no lobbying' but under lobbying as well.

Like for  $SW$  and  $GW(l_I : c(l_I) = 0)$ , we have now shown the conditions under which the effect of  $\phi$  on  $GW(l_I)$  is monotonic. So we can infer the effect on  $GW(l_I)$  of adoption of naturalization, formalized by an increase in  $\phi$  from  $\phi = 0$  to  $\phi = \bar{\phi}$ . If  $w^* + \gamma < \underline{\varpi}^{NB}$  then  $\partial GW(l_I)/\partial \phi > 0$  for any  $\phi \in [0, \bar{\phi}]$ , and so  $GW(l_I)$  must increase for an increase of  $\phi$  from  $\phi = 0$  to  $\phi = \bar{\phi}$ . If on the other hand  $w^* + \gamma > \overline{\varpi}^{NB}$  then  $\partial GW(l_I)/\partial \phi < 0$  for any  $\phi \in [0, \bar{\phi}]$ , and so  $GW(l_I)$  must decrease for such an increase in  $\phi$ .

We will see in due course that  $\sigma$  plays a decisive role in the determination of whether the government's payoff is maximized under naturalization,  $\phi = \bar{\phi}$ , or without naturalization,  $\phi = 0$ . But first, to establish a parallel between our analysis of GWL with GNL, let us fix government bargaining



power at its minimum level,  $\sigma = 0$ .

To make comparisons between the planner's solution, GNL and GWL, we now broaden our definition of  $\bar{w}$  to incorporate  $\bar{w}^{NB}$  as follows.

$$\text{Let } \underline{w} = \min \{ \underline{w}^{SW}, \underline{w}^{NL}, \underline{w}^{NB} \}, \text{ and } \bar{w} = \max \{ \bar{w}^{SW}, \bar{w}^{NL}, \bar{w}^{NB} \}.$$

Then we can say for sure that if  $w^* + \gamma < \underline{w}$ , then  $\partial SW / \partial \phi > 0$ ,  $\partial GW(l_I : c(l_I) = 0) / \partial \phi > 0$  and  $\partial GW(l_I) / \partial \phi > 0$  for all  $\phi \in [0, \bar{\phi}]$ . And if  $w^* + \gamma > \bar{w}$ , then  $\partial SW / \partial \phi < 0$ ,  $\partial GW(l_I : c(l_I) = 0) / \partial \phi < 0$  and  $\partial GW(l_I) / \partial \phi < 0$  for all  $\phi \in [0, \bar{\phi}]$ .

The next result draws a direct parallel between Proposition 1 for the social planner's solution, Proposition 2 for GNL, and GWL.

**Proposition 6:** Fix  $z = 1$  and  $\sigma = 0$ . GWL exhibits the following features.

(i) The immigration quota, level of capital, profit, net income of capital owners, and quota rent, are all increasing in the naturalization rate,  $\phi$ , while the domestic wage is decreasing in the naturalization rate.

(ii) The government's payoff,  $GW(l_I)$ , is strictly convex in the naturalization rate,  $\phi$ , and monotonically increasing (decreasing) in  $\phi$  for immigrant wage  $w^* + \gamma < \underline{w}^{NB}$  ( $w^* + \gamma > \bar{w}^{NB}$ ).

(iii) If  $w^* + \gamma < \underline{w}$  then, all else equal, the government's payoff is maximized if its hands are tied to naturalization,  $\phi = \bar{\phi}$ , and this maximizes social welfare. If  $w^* + \gamma > \bar{w}$  then, all else equal, the government's payoff is maximized if its hands are not tied to naturalization,  $\phi = 0$ , and this maximizes social welfare.

Parts (i) and (ii) of Proposition 6 directly parallel parts (i) and (ii) of Propositions 1 and 2. This illustrates the commonalities between the social planner's problem and that of GNL and GWL. Part (iii) is a direct parallel to part (iii) of Proposition 2 and similar to Part (iii) of Proposition 1: it shows that, like for social welfare and GNL, when the immigrant wage is sufficiently low, the government's payoff will be maximized if it is constrained by naturalization, and this maximizes social welfare. On the other hand, when the immigrant wage is sufficiently high, the government's payoff will be maximized if its hands are not tied to naturalization, and this maximizes social welfare. Under GWL, like GNL but unlike the planner, the government does not get to choose the naturalization rate because it is set exogenously, and by assumption does not get to choose whether or not to adopt naturalization.

Next, we consider specific economic outcomes under GWL and how these compare to those

under the planner's solution.

**Proposition 7:** Fix  $z = 1$ , and  $\sigma = 0$  and assume either that  $w^* + \gamma < \underline{\omega}$ , or that  $w^* + \gamma > \bar{\omega}$ . Because capital owners are able to mitigate the hold-up problem through lobbying, the immigration quota, profit, and the net income to capital owners are higher under GWL than under GNL, but the domestic wage is lower under GWL than GNL. Overall, because the government could refuse any contribution offered by the lobby, the government payoff is higher under GWL than GNL.

Proposition 7 draws a parallel to Proposition 3 under GNL. It says that the level of capital, immigration, profits, and net income of capital owners are higher under GWL than GNL because capital owners are able to mitigate the hold-up problem through lobbying. This holds regardless of whether  $w^* + \gamma$  is sufficiently low, or sufficiently high, to ensure that both the social welfare function and the government's payoff function are monotonically increasing, or monotonically decreasing, in  $\phi$ . The fact that capital owners can mitigate the hold-up problem through lobbying means that both they and the government are better off under GNL than under GWL. Workers suffer lower wages under GWL than GNL as a result of the fact that the immigration quota is higher under GWL.

We are now in a position to examine how the government's payoff to naturalization changes as a result of a change in the government's bargaining power. To do this, using (27), we differentiate  $\partial GW(l_I) / \partial \phi$  with respect to  $\sigma$  to obtain

$$\frac{\partial^2 GW(l_I)}{\partial \phi \partial \sigma} = \frac{\psi^{NB} \partial \beta / \partial \sigma + (1 - 2\alpha) \beta \partial \psi^{NB} / \partial \sigma}{(1 - 2\alpha)^2 (1 - \phi) \beta} K^{NB}, \quad (28)$$

where

$$\beta = 2 + zs(2 + a(2 - zs)(1 - \sigma)). \quad (29)$$

By inspecting (26) and (29), we observed that  $\psi^{NB}$  is increasing in  $\sigma$  while  $\beta$  is decreasing in  $\sigma$ . It follows by inspection of (28) that, for  $\alpha$  sufficiently close to  $\frac{1}{2}$ ,  $|\psi^{NB} \partial \beta / \partial \sigma| > (1 - 2\alpha) \beta \partial \psi^{NB} / \partial \sigma$  and so  $\partial^2 GW(l_I) / \partial \phi \partial \sigma < 0$ . So (28) tells us is that  $\partial GW(l_I) / \partial \phi$  is decreasing in  $\sigma$  for  $\alpha$  sufficiently close to  $\frac{1}{2}$ . Now recall that by Proposition 6,  $\partial GW(l_I) / \partial \phi > 0$  for immigrant wage  $w^* + \gamma < \underline{\omega}^{NB}$ , given that  $\sigma = 0$ . By (28), providing that  $\alpha$  is sufficiently close to  $\frac{1}{2}$  and hence  $\partial^2 GW(l_I) / \partial \phi \partial \sigma$  is sufficiently large in absolute magnitude, there must exist a threshold value of  $\sigma \in (0, 1)$  at which  $\partial GW(l_I) / \partial \phi = 0$ . We will call this threshold value  $\bar{\sigma}$ . By the fact that  $\partial^2 GW(l_I) / \partial \phi \partial \sigma < 0$ , we have that  $\partial GW(l_I) / \partial \phi > 0$  for  $\sigma \in [0, \bar{\sigma})$ , and  $\partial GW(l_I) / \partial \phi < 0$  for  $\sigma \in (\bar{\sigma}, 1]$ .

Therefore, even though for an interval  $\sigma \in [0, \bar{\sigma})$ , the government gains from an increase in the

naturalization rate  $\phi$ , for higher levels of government bargaining power  $\sigma \in (\bar{\sigma}, 1]$  this reverses, and the government suffers a decrease in its payoff from an increase in  $\phi$ . What this means in turn is that for relatively weak bargaining power,  $\sigma \in [0, \bar{\sigma})$ , the government would benefit from having its hands tied to the maximal naturalization rate,  $\phi = \bar{\phi}$ , while for relatively strong bargaining power it would benefit from having no naturalization at all,  $\phi = 0$ .

The solution for investment,  $K^{NB}$ , holds the key to understanding this result. As we saw in the discussion following (24),  $K^{NB}$  is decreasing in  $\sigma$ : the weaker is the government's bargaining power, the more lobbying mitigates the hold-up problem. For  $\sigma \in [0, \bar{\sigma})$ , not only is investment higher. In addition, the government actually gains from an increase in the naturalization rate,  $\phi$ , through the higher level of investment, even though this reduces the amount of revenue it gains in quota rent and worsens the domestic wage. Hence why the government's payoff is maximized at  $\phi = \bar{\phi}$ . When  $\sigma \in (\bar{\sigma}, 1]$ , on the other hand, higher investment from higher  $\phi$  is not high enough to offset the counterveiling forces of lower quota rent and a lower domestic wage. Hence, in that case, the government's payoff is maximized at  $\phi = 0$ .

We can go one step further and determine the implications for social efficiency of variation in  $\sigma$ . To do so, we need to replace  $\underline{\varpi}^{NB}$  in the above analysis with  $\underline{\varpi}$ . Then, if we assume that  $w^* + \gamma < \underline{\varpi}$ , we are in the range where social welfare, GNL and GWL are all monotonically increasing in the naturalization rate,  $\phi$ . We can now say that for  $\sigma \in [0, \bar{\sigma})$ , not only is the government's payoff maximized at the naturalization rate  $\phi = \bar{\phi}$ , but social welfare is as well. On the other hand, if  $\sigma \in (\bar{\sigma}, 1]$ , the government's payoff is maximized when there is no naturalization  $\phi = 0$  while social welfare is maximized at the maximal rate,  $\phi = \bar{\phi}$ .

Our discussion so far is summarized as follows.

**Proposition 8:** Fix  $z = 1$ ,  $w^* + \gamma < \bar{\varpi}$ ,  $\alpha \rightarrow \frac{1}{2}$ , and allow  $\sigma \in [0, 1]$ . There exists a unique threshold value,  $\bar{\sigma} \in (0, 1)$  such that:

(i) for  $\sigma < \bar{\sigma}$  the government's payoff is maximized if its hands are tied to the socially efficient naturalization rate,  $\phi = \bar{\phi}$ ;

(ii) for  $\sigma > \bar{\sigma}$ , the government's payoff is maximized if its hands are not tied to naturalization,  $\phi = 0$ , while the socially efficient naturalization rate is  $\phi = \bar{\phi}$ .

This result shows how varying  $\sigma$  can overturn part (ii) of Proposition 6. Part (i) of Proposition 8 shows that Part (ii) of Proposition 6 generalizes from  $\sigma = 0$  to  $\sigma < \bar{\sigma}$ . Part (ii) of Proposition 8

shows that part (ii) of Proposition 6 is overturned for the interval  $\sigma > \bar{\sigma}$ . It is significant that over this interval of  $\sigma$ , the government's payoff does not coincide with the socially efficient naturalization rate. Specifically, if government bargaining power is sufficiently strong then it gains more from being able to extract rents from capital owners, even though this undermines investment. So the government is better off under  $\phi = 0$  where it can maximize the rents it extracts for relaxing the immigration quota, even though social welfare would be maximized at  $\phi = \bar{\phi}$ .

Notice that Proposition 8 also relies on a sufficiently high value of  $\alpha$  because this in turn implies that, with capital share in the production process being relatively high, capital generates sufficient rents for extraction by the government through the lobbying process. If this did not hold then the damage to investment through the hold-up problem would actually lead to a decline in the government's payoff and rent extraction would not be sufficient to compensate for this.

#### 5.4 Dictatorship and Democracy with Lobbying

In Section 4.4, where we compared dictatorship to democracy with 'no lobbying', our sole differentiator between democracy and dictatorship was through variation of the parameter  $z$ , between  $z = 1$  for democracy and  $z > 1$  for dictatorship. In this section, we incorporate the idea that government bargaining power is likely to be stronger under dictatorship than under democracy, as discussed in the Introduction. In the simplest terms, we will characterize democracy as  $z = 1$  and  $\sigma < \bar{\sigma}$ , and dictatorship as  $z > 1$  and  $\sigma > \bar{\sigma}$ . This is different to our analysis of the previous subsection because there we were maintaining the assumption that  $z = 1$  throughout, while allowing  $\sigma$  to vary. With  $z \geq 1$  we will maintain our restriction on  $s$  that  $s < (1 - 2\phi)/z$ .

The natural question that follows is whether naturalization is more likely to lead to an increase in the government payoff under dictatorship or democracy. We can consider this by examining the effect of an increase in  $z$  on  $\partial GW(l_I : c(l_I) = 0) / \partial \phi$ . Differentiating (27) with respect to  $z$ , we obtain

$$\frac{\partial^2 GW(l_I)}{\partial \phi \partial z} = \frac{\chi \psi^{NB}}{(1 - 2\alpha)^2 (1 - \phi) \beta} K^{NB},$$

where

$$\begin{aligned} \chi &= (1 + a) s \beta + (1 + (1 + a) z s) \frac{\partial \beta}{\partial z} \\ &= 2s(1 + a(1 - zs)(1 - \sigma)) > 0. \end{aligned}$$

We can see by inspection that  $\partial^2 GW(l_I) / \partial \phi \partial z > 0$ . Therefore, by the same argument as for GWL,

$\partial GW(l_I) / \partial \phi$  is increasing in  $z$  not just for small increases in  $z$  but for a discrete increase in  $z$  to  $z' > z$ . We then have an equivalent result for  $GW(l_I)$  under GWL, to Lemma 1 that was established for  $GW(l_I : c(l_I) = 0)$  under GNL. However, for the purposes of our current discussion, it will be helpful to modify the result. For this purpose, let  $\bar{z}$  be the value of  $z$  for which  $\partial GW(l_I) / \partial \phi = 0$ . Then we have the following

**Lemma 3:** *Assume an initial set of parameters, including  $z = 1$ , for which  $\partial GW(l_I) / \partial \phi < 0$ . Then there exists a value,  $\bar{z} > 1$  for which  $\partial GW(l_I) / \partial \phi = 0$ , with  $\partial GW(l_I) / \partial \phi > 0$  for all  $z > \bar{z}$ .*

Lemma 3 shows that, like for GNL, under GWL there always exists a value  $z = \bar{z}$  sufficiently large that, all else equal, the government's payoff becomes increasing in the naturalization rate,  $\partial GW(c(l_I) = 0) / \partial \phi > 0$ , for  $z > \bar{z}$ , even if at  $z = 1$  it is the case that  $\partial GW(c(l_I) = 0) / \partial \phi < 0$ .

Now, while continuing to think of dictatorship as a country for which  $z > 1$ , we will think of it as being represented by  $z' \in (1, \bar{z})$ , as opposed to thinking of it as a country for which  $z' > \bar{z}$  as we did in Proposition 4. So while we will continue to think of a dictatorship as a country for which  $z > 1$ , it will not be the case that  $\partial GW(l_I) / \partial \phi > 0$  because of the value of  $z$ . In our present set-up, if  $\partial GW(l_I) / \partial \phi > 0$  it will be because  $\sigma > \bar{\sigma}$ .

We can now extend Proposition 8 by relaxing the restriction that  $z = 1$  to one where  $z \in [1, \bar{z}]$ , as follows.

**Proposition 9:** *Fix  $w^* + \gamma < \bar{w}$ ,  $\alpha \rightarrow \frac{1}{2}$ , and allow  $z \in [1, \bar{z}]$ ,  $\sigma \in [0, 1]$ . There exists a unique threshold value,  $\bar{\sigma} \in (0, 1)$  such that:*

- (i) *for a democracy, characterized by  $\sigma < \bar{\sigma}$  and  $z = 1$ , the government's payoff is maximized if its hands are tied to the socially efficient naturalization rate,  $\phi = \bar{\phi}$ ;*
- (ii) *for a dictatorship, characterized by  $\sigma > \bar{\sigma}$  and  $z' \in (1, \bar{z})$ , the government's payoff is maximized if its hands are not tied to naturalization,  $\phi = 0$ , while the socially efficient naturalization rate is  $\phi = \bar{\phi}$ .*

This result shows that Proposition 8 extends to a situation where democracy and dictatorship are differentiated by both  $\sigma$  and  $z$ . It says that, providing  $z'$  is not too large, i.e.  $z' \in (1, \bar{z})$ , it is the higher bargaining power of dictatorships that drives their unwillingness to be constrained by naturalization, even though this would be more efficient for the country as a whole. A sufficiently large value for  $z'$  could overturn this tendency. That is, setting  $z' > \bar{z}$  would restore the feature of Proposition

4 whereby  $GW(l_I)$  is maximized at  $\phi = \bar{\phi}$  under dictatorship. But the fact we observe dictatorships not being constrained by naturalization in practice suggests considerations around bargaining power may dominate. Proposition 9 is the first main component in our argument for rationalizing the Koopmans-Michalowski paradox, that democracies have a greater tendency to extend immigration rights through naturalization than dictatorships have.

The second component of the argument is to explain why dictatorships may nevertheless set more open immigration policy. To explore this, we want to examine the government's incentive to adjust the immigration quota,  $l_I^{NB}$ , in response to a change in the two differentiators between dictatorship and democracy,  $\sigma$  and  $z$ , as well as the fact that a democracy is more likely to be constrained by naturalization while a dictatorship is not. Our aim will be to show that while a higher level of  $\sigma$  and a lower level of  $\phi$  associated with dictatorship creates a tendency towards a lower level of  $l_I^{NB}$ , the higher level of  $z$  that is also associated with dictatorship can more than offset this, bringing about a higher level of  $l_I^{NB}$  under dictatorship than democracy.

We now want to develop a framework to examine how  $l_I^{NB}$  changes in response to a move from democracy to dictatorship. For this purpose, assume a set of underlying parameters including  $z = 1$  and  $\phi = \bar{\phi} > 0$  such that  $\sigma = \bar{\sigma}$  exists. Take the equilibrium outcome associated with this set of parameters to represent democracy. Now let  $z' > 1$ ,  $\phi = 0$ , and  $\sigma' > \bar{\sigma}$  represent the equilibrium outcome under dictatorship. And let  $dz = z' - z$ ,  $d\phi = -\bar{\phi}$ , and  $d\sigma = \sigma' - \bar{\sigma}$ . Now obtain the reduced form of  $l_I^{NB}$  by substituting (22) and (24) into 21). By taking the total derivative of the resulting expression, we can then measure the change of  $l_I^{NB}$  in response to the move from democracy to dictatorship,  $dl_I^{NB}$ , as

$$dl_I^{NB} = \frac{\partial l_I^{NB}}{\partial z} dz + \frac{\partial l_I^{NB}}{\partial \phi} d\phi + \frac{\partial l_I^{NB}}{\partial \sigma} d\sigma. \quad (30)$$

To show that  $l_I^{NB}$  under dictatorship is higher than under democracy, we need to show that  $dl_I^{NB} > 0$ .

The partial derivatives in (30) can be obtained from the reduced form expression for  $l_I^{NB}$ . Recognizing that  $w^{NB}$  is a function of  $z$  only, while  $K^{NB}$  is a function of  $\sigma$  and  $z$ , we have:

$$\frac{\partial l_I^{NB}}{\partial z} = \left( \frac{1}{2(w^{NB})^3} \right) (K^{NB})^{2\alpha-1} \left( -K^{NB} \frac{\partial w^{NB}}{\partial z} + \alpha w^{NB} \frac{\partial K^{NB}}{\partial z} \right) > 0, \quad (31)$$

because  $\partial w^{NB}/\partial z < 0$  by inspection of (22), and  $\partial K^{NB}/\partial z > 0$ : see the discussion following (24).

Similarly,

$$\frac{\partial l_I^{NB}}{\partial \phi} = \left( \frac{1}{2(w^{NB})^3} \right) (K^{NB})^{2\alpha-1} \left( -K^{NB} \frac{\partial w^{NB}}{\partial \phi} + \alpha w^{NB} \frac{\partial K^{NB}}{\partial \phi} \right) > 0 \quad (32)$$

because  $\partial w^{NB}/\partial\phi < 0$  by inspection of (22), and  $\partial K^{NB}/\partial\phi > 0$ : again, see the discussion following (24). On the other hand,

$$\frac{dl_I^{NB}}{d\sigma} = \left( \frac{1}{2(w^{NB})^2} \right) \alpha (K^{NB})^{2\alpha-1} \frac{\partial K^{NB}}{\partial\sigma} < 0 \quad (33)$$

again based on the discussion following (24).

Equations (32) and (33) show that, holding  $z$  constant,  $dl_I^{NB} < 0$  because  $d\phi = -\bar{\phi} < 0$  and  $\partial l_I^{NB}/\partial\phi > 0$ , while  $d\sigma > 0$  and  $\partial l_I^{NB}/\partial\sigma < 0$ . Hence, if only  $\phi$  and  $\sigma$  were different between the democracy and the dictatorship under consideration, then we would have more restrictive immigration policy under dictatorship than under democracy. However, if we do allow  $z$  to increase, the fact that  $\partial l_I^{NB}/\partial z > 0$  means we can always find a value of  $z'$  under dictatorship sufficiently large, and in turn  $dz$  sufficiently large, that  $dl_I^{NB} > 0$ . Hence, for  $dz$  sufficiently large, the immigration quota under dictatorship is larger than under democracy, which is the second component required to resolve the Koopmans-Michalowski paradox.

To complete the discussion, we also need to address the question of whether the value of  $z'$  required for  $dl_I^{NB} > 0$  is in the range  $z' \in (1, \bar{z})$ . In general, it may be that (the absolute value of)  $d\phi$  and  $d\sigma$  are sufficiently large that the value of  $z'$  required for  $dl_I^{NB} > 0$  would imply  $z' > \bar{z}$ . This would create a tendency for the dictatorship to benefit from naturalization. However, if we say that  $-\bar{\phi} \rightarrow 0$  and  $\sigma' \rightarrow \bar{\sigma}$ , and hence  $d\phi$  and  $d\sigma$  tend to zero, then by (31)-(33), and in particular by the continuity of these functions in their respective arguments, we can find a value of  $z' \in (1, \bar{z})$  such that  $dl_I^{NB} > 0$ . We can summarize the foregoing discussion as follows.

**Proposition 10:** Fix  $w^* + \gamma < \bar{w}$ ,  $\alpha \rightarrow \frac{1}{2}$ , and allow  $\sigma \in [0, 1]$ . Assume a set of parameters for which there is a democracy, represented by  $\sigma = \bar{\sigma}$ , and  $z = 1$ , and  $\phi = \bar{\phi} > 0$ , and a dictatorship represented by  $\sigma' > \bar{\sigma}$ ,  $\phi = 0$ , and  $z' > 1$ , whereby  $-d\phi \rightarrow 0$  and  $d\sigma \rightarrow 0$ . Then there exists a value of  $z' \in (1, \bar{z})$  such that  $dl_I^{NB} > 0$ .

We interpret this result as follows. An increase in bargaining power is sufficient to reduce  $K^{NB}$  through the hold-up problem, and this in turn will reduce  $l_I^{NB}$ . An absence of naturalization under dictatorship means that there is no mitigation of the hold-up problem as there would be under democracy, and this also reduces  $l_I^{NB}$ . But if a dictatorship cares enough about capital owners, reflected in a higher level of  $z$ , then this will be sufficient to offset the negative effect of the hold-up problem on  $K^{NB}$ , and hence  $l_I^{NB}$  is higher under dictatorship than under democracy.

However, Proposition 10 highlights an important qualification to this argument. Government bargaining power cannot be too much greater under dictatorship, and the rate of naturalization cannot be too much higher under democracy. If they were, then the additional weight that the government would have to put on the income of capital owners would have to be so large that the dictatorship would end up gaining from naturalization. But that appears to be counterfactual.

Taking Propositions 9 and 10 together, we have now provided a possible resolution to the Koopmans-Michalowski paradox. Proposition 9 explains how a dictatorship could be better off not tying its hands to naturalization, while a democracy would be better off under naturalization. At the same time, Proposition 10 explains the conditions under which the dictatorship sets its immigration quota at a higher level than the democracy because it places a greater weight on the welfare of capital owners. We have now confirmed that the introduction of lobbying into our model can then predict that while dictatorships are more open when it comes to immigration policy, democracies have a greater tendency to extend naturalization rights to immigrants.

We can now return to our assumption that dictatorships tend to be capitalist: that is, they tend to care more about the welfare of capital owners than workers. We have embodied this in our assumption that  $z > 1$ . Our analysis further reveals how a communist dictatorship, that does not favor the income of capitalists and might even disfavor them by setting  $z < 1$ , would tend to have a lower immigration quota than a democracy. Thus, our model can explain why China, a communist dictatorship, has such tight immigration policy, reflected in just 0.1% of its population being immigrants.<sup>22</sup>

## 6 Conclusion

This paper makes two main contributions. It presents the first economics-based characterization of naturalization, based on the solution to the planner's problem. This theory could usefully form a basis for future studies of the economics of naturalization. In the same way as free trade forms a reference point in the study of international trade policy, the solution to the planner's problem could form a reference point in studies of naturalization. At the same time, our theory shows that naturalization may not necessary increase social welfare. Future empirical work could usefully examine the effect of naturalization rights on social welfare. One of our key results is that naturalization only increases social welfare if the immigrant wage is sufficiently low. This raises the intriguing question of whether naturalization might have increased social welfare in the past, when the immigrant wage

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<sup>22</sup><https://www.un.org/en/development/desa/population/migration/data/estimates2/countryprofiles.asp>



was relatively low, but whether if the immigrant wage has been rising over time, naturalization rights might now have a damaging effect on social welfare.

The paper's second contribution is to resolve what we call, 'the Koopmans-Michalowski paradox'. The past literature on the political economy of immigration policy has tended to consider the proximate influences of voters on the one hand and lobby contributions on the other. This is true at a theoretical level (Facchini and Willmann 2005) and an empirical level (Facchini et al 2011). Our resolution to the Koopmans-Michalowski paradox suggests that the political economy of immigration policy may be more far-reaching than previously considered, linking the government's incentive to hold up capital owners over their investment decisions. The theoretical framework developed in the present paper could be used to inform future econometric investigations that would take into account the impact of immigration policy decisions on investment, as mediated through the prevailing institutional setting. Gawande and Jo (2014) propose a method for doing this in the context of the MR-C model of international trade agreements. This could be extended in a natural way to consider the effects of naturalization.

A further insight that emerges from the analysis of this paper is that for the government, allowing lobbying on the one hand, and tying its hands against lobbying on the other, can be alternative routes to economic efficiency. This challenges a prevailing assumption of the literature on international trade agreements that lobbying tends to reduce economic efficiency. The prevailing assumption has been used to motivate the 'commitment-based theory' of trade agreements: that a government uses a trade agreement to tie its hands against the temptation to engage in efficiency-reducing lobbying. In our model, economic efficiency is actually enhanced when capital owners lobby to reduce immigration policy because this leads to a reduction of the distortion caused by the policy. We can nevertheless disentangle these competing effects to show that a dictatorship does better in the absence of naturalization, even though naturalization would be efficiency enhancing.

This insight opens the door to an exploration of the same underlying tensions over policies and institutions in other areas. For example, dictatorships tend to welcome foreign direct investment, but fail to fully support contract enforcement through the courts by which the government could tie its hands to the protection of property rights of foreigners that would increase capital formation and economic efficiency. The underlying tensions are aligned with those that we consider in the present paper and the opposite of those in international trade policy. They could be analyzed using a similar approach to the one that we have developed to study immigration in the present paper.

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## A Appendix

### A.1 Derivations

#### Reduced form of Social Welfare Function and Government Payoff Function

Beginning with the social welfare function, (4), use (3) to substitute for  $\Pi(l_I)$ , and use (6) to substitute for  $l_I$  at the efficient level,  $l_I^{SP}$ :

$$SW = (1 - \phi)(w^* + \gamma) + \frac{(1 + s)w - (1 - \phi)(w^* + \gamma)}{4w^2} K^{2\alpha} - sK.$$

Next, use (7) to substitute for  $w$  at the efficient level,  $w^{SP}$ :

$$SW = \frac{1 + s}{2} w^{SP} + \frac{1}{8} \frac{1 + s}{w^{SP}} K^{2\alpha} - sK.$$

Finally, use (8) to substitute for  $K$  at the efficient level,  $K^{SP}$ :

$$SW = \frac{1 + s}{2} w^{SP} + \frac{s}{2\alpha} (K^{SP})^{1-2\alpha} (K^{SP})^{2\alpha} - s(K^{SP}).$$

Then simplify to obtain (11).

Based on (11), the expression for  $\partial SW / \partial \phi$  is calculated as

$$\frac{\partial SW}{\partial \phi} = \frac{1 + s}{2} \frac{\partial w^{SP}}{\partial \phi} + \psi^{SP} \frac{\partial K^{SP}}{\partial \phi},$$

where

$$\frac{\partial w^{SP}}{\partial \phi} = -\frac{1}{1 - \phi} w^{SP} \text{ and } \frac{\partial K^{SP}}{\partial \phi} = \frac{1}{(1 - 2\alpha)(1 - \phi)} K^{SP}.$$

Substituting and simplifying yields (12).

Since the government's problem takes the same basic form as the social planner's problem, the same steps can be used on the corresponding functions in the case of GNL to derive (17) for  $GW(l_I : c(l_I) = 0)$  and 19) for  $\partial GW(l_I : c(l_I) = 0) / \partial \phi$ . These steps can also be used in the case of GWL to derive (25) for  $GW(l_I)$  and (27) for  $\partial GW(l_I) / \partial \phi$ .

## A.2 Proofs

### Proof of Lemma 2

Using using (1) and (5) in (20), we obtain the following reduced form:

$$\begin{aligned}
JS &= GW(l_I) + a(s(\pi(l_I) + r(l_I)K - K) - c(l_I)l_I) \\
&= zs(\pi(l_I) + r(l_I)K - K) + w(l_I)1 + (w(l_I) - w^* - \gamma)l_I + azs(\pi(l_I) + r(l_I)K - K) \\
&= \frac{zs}{2}K^\alpha\sqrt{1+l_I} + \frac{1}{2}K^\alpha\sqrt{1+l_I} - (w^* + \gamma)l_I + \frac{azs}{2}K^\alpha\sqrt{1+l_I} - zs(1+a)K \\
&= \frac{zs(1+a)}{2}K^\alpha\sqrt{1+l_I} + \frac{1}{2}K^\alpha\sqrt{1+l_I} - (w^* + \gamma)l_I - zs(1+a)K
\end{aligned} \tag{34}$$

The government and the lobby share the surplus according to their bargaining powers-

$$\begin{aligned}
B &= \left( SW^{NL}(l_I) - SW^{NB}(l_I^{NB}) - ac(l_I^{NB})l_I^{NB} \right)^\sigma \\
&\quad \left( \pi(l_I^{NB}) + r(l_I^{NB})K - K - c(l_I^{NB})l_I^{NB} - \pi(l_I^{NL}) - r(l_I^{NL})K + K \right)^{1-\sigma}
\end{aligned} \tag{35}$$

This yields

$$c(l_I^{NB})l_I^{NB} = \frac{(1-\sigma)(SW^{NL}(l_I^{NL}) - SW^{NB}(l_I^{NB}))}{a} + \sigma \left( \pi(l_I^a) + r(l_I^a)K - \pi(l_I^{NL}) - r(l_I^{NL})K \right) \tag{36}$$

To derive the optimal contribution schedule, we will analyse each component separately,

$$\begin{aligned}
SW^{NL}(l_I^{NL}) &= zs \left( \pi(l_I^{NL}) + r(l_I^{NL})K - K \right) + w(l_I^{NL}) + (w(l_I^{NL}) - w^* - \gamma)l_I^{NL} \\
&= zs \left( \left( \frac{1}{2} - \alpha \right) K^\alpha \sqrt{1+l_I^{NL}} + \alpha K^{\alpha-1} \sqrt{1+l_I^{NL}}K \right) + \frac{1}{2} \frac{K^\alpha}{\sqrt{1+l_I^{NL}}} \\
&\quad + \left( \frac{1}{2} \frac{K^\alpha}{\sqrt{1+l_I^{NL}}} - w^* - \gamma \right) l_I^{NL} - zsK \\
&= \left( \frac{zs+1}{2} \right) K^\alpha \sqrt{1+l_I^{NL}} - (1-\phi)(w^* + \gamma)l_I^{NL} - zsK
\end{aligned} \tag{37}$$

Similarly,

$$SW^{NB}(l_I^{NB}) = \left( \frac{zs+1}{2} \right) K^\alpha \sqrt{1+l_I^{NB}} - (1-\phi)(w^* + \gamma)l_I^{NB} - zsK \tag{38}$$

Combining equation (37) and (38),

$$SW^{NL}(l_I^{NL}) - SW^{NB}(l_I^{NB}) = \left(\frac{k+1}{2}\right) K^\alpha \left[ \sqrt{1+l_I^{NL}} - \sqrt{1+l_I^{NB}} \right] - (w^* + \gamma)(l_I^{NL} - l_I^{NB}) \quad (39)$$

Considering the second term of (36),

$$\left( \pi(l_I^{NB}) + r(l_I^{NB})K - \pi(l_I^{NL}) - r(l_I^{NL})K \right) = \frac{1}{2} K^\alpha \left[ \sqrt{1+l_I^{NB}} - \sqrt{1+l_I^{NL}} \right] \quad (40)$$

Substituting (39) and (40) into the contribution schedule expressed in (36),

$$\begin{aligned} c(l_I^{NB})l_I^{NB} &= \left(\frac{1-\sigma}{a}\right) \left[ \left(\frac{zs+1}{2}\right) K^\alpha \left[ \sqrt{1+l_I^{NL}} - \sqrt{1+l_I^{NB}} \right] - (w^* + \gamma)(l_I^{NL} - l_I^{NB}) \right] \\ &\quad + \frac{\sigma}{2} K^\alpha \left[ \sqrt{1+l_I^{NB}} - \sqrt{1+l_I^{NL}} \right] \end{aligned} \quad (41)$$

Thus, the contribution schedule simplifies to

$$\begin{aligned} c(l_I^{NB})l_I^{NB} &= K^\alpha \left[ \sqrt{1+l_I^{NL}} - \sqrt{1+l_I^{NB}} \right] \left[ \left(\frac{1-\sigma}{a}\right) \left(\frac{1+zs}{2}\right) - \frac{\sigma}{2} \right] \\ &\quad - \left(\frac{1-\sigma}{a}\right) (w^* + \gamma)(l_I^{NL} - l_I^{NB}) \end{aligned}$$

From the derivation of optimal immigration when  $a = 0$  and  $a > 0$ , we have already obtained the values for  $l_I^{NL}$  and  $l_I^{NB}$ .

$$K^\alpha \left[ \sqrt{1+l_I^{NL}} - \sqrt{1+l_I^{NB}} \right] = K^\alpha \left[ \left(\frac{1+zs}{4}\right) \left(\frac{K^\alpha}{w^* + \gamma}\right) - \frac{K^\alpha}{4(w^* + \gamma)} (1 + zs(1+a)) \right] \quad (42)$$

Consequently, the first term of (41) becomes

$$\begin{aligned} &K^\alpha \left[ \sqrt{1+l_I^{NL}} - \sqrt{1+l_I^{NB}} \right] \left[ \left(\frac{1-\sigma}{a}\right) \left(\frac{1+zs}{2}\right) - \frac{\sigma}{2} \right] \\ &= \frac{K^{2\alpha}}{4(w^* + \gamma)} \left[ (1+zs) - (1+zs(1+a)) \right] \left[ \left(\frac{1-\sigma}{a}\right) \left(\frac{1+zs}{2}\right) - \frac{\sigma}{2} \right] \\ &= -\frac{zsK^{2\alpha}}{8(w^* + \gamma)} \left[ 1 + zs(1-\sigma) - \sigma(1+a) \right] \end{aligned} \quad (43)$$

Similarly, plugging in the values of  $l_I^{NL}$  and  $l_I^{NB}$  into the second term of (41) yields,

$$\begin{aligned} \left(\frac{1-\sigma}{a}\right)(1-\phi)(w^*+\gamma)(l_I^{NL}-l_I^{NB}) &= \left(\frac{1-\sigma}{a}\right)\frac{K^{2\alpha}}{16(1-\phi)(w^*+\gamma)^2}\left[(1+s)^2-[1+s(1+a)]^2\right] \\ &= -\frac{K^{2\alpha}(1-\sigma)}{16(1-\phi)(w^*+\gamma)}[2zs+2z^2s^2+az^2s^2] \end{aligned}$$

Plugging in (42) and (43) into (41) and simplifying

$$\begin{aligned} c(l_I^{NB})l_I^{NB} &= -zs\frac{K^{2\alpha}}{8(1-\phi)(w^*+\gamma)}\left[1+zs(1-\sigma)-\sigma(1+a)\right]+\frac{K^{2\alpha}(1-\sigma)}{16(1-\phi)(w^*+\gamma)}\left[2zs+2z^2s^2+az^2s^2\right] \\ &= \frac{K^{2\alpha}}{16(1-\phi)(w^*+\gamma)}\left[2zsa\sigma+az^2s^2-\sigma az^2s^2\right] \\ &= \frac{azs(2\sigma+zs(1-\sigma))}{16(1-\phi)(w^*+\gamma)}K^{2\alpha} \end{aligned}$$

This is (23), and represents the optimal contribution schedule that the lobby is willing to provide the government to persuade the government to increase its immigration quota. The result follows.  $\square$

### Proof of Proposition 5

Here we present a detailed derivation of optimal investment by capital owners in Stage 1. The net income of the capital owners under political equilibrium is given by

$$\Pi(l_I^{NB}) = \pi(l_I^{NB}) + r(l_I^{NB})K - K - c(l_I^{NB})l_I^{NB}$$

Substituting the values for immigration and contributions from equations 21 and 23,

$$\Pi(l_I^{NB}) = \frac{1}{2}\frac{K^{2\alpha}}{4(w^*+\gamma)}[1+zs(1+a)] - \frac{zsK^{2\alpha}}{16(w^*+\gamma)}\left[2a\sigma+azs(1-\sigma)\right] - K$$

The first order condition for maximization is

$$\frac{\partial \Pi(l_I^{NB})}{\partial K} = 0 \Rightarrow \frac{\alpha K^{1-2\alpha}}{8(w^*+\gamma)}\left[2+2zs(1+a)-2zsa\sigma-az^2s^2(1-\sigma)\right] = 1$$

From this, optimal capital investment is

$$K^{NB} = \left[\frac{\alpha}{8(w^*+\gamma)}\left[2(1+zs(1+a))-zs[2a\sigma+azs(1-\sigma)]\right]\right]^{\frac{1}{1-2\alpha}}$$

Equation (24) is obtained using (22).  $\square$